This Number Completes Volume 8
VOLUME 8, NUMBER 6, DECEMBER 1960

THE JOURNAL OF THE AMERICAN NUCLEAR SOCIETY

# NUCLEAR SCIENCE and ENGINEERING

ACADEMIC PRESS
NEW YORK and LONDON

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VOLUME 8, NUMBER 6, December 1960

NUCLEAR SCIENCE AND ENGINEERING: 8, 670–690 (1960)

# General Solution of the Reactor Kinetic Equations*

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*Received August 22, 1960*

The reactor kinetic equations are reduced to an integral form convenient for explicit numerical solution, involving no approximations beyond the usual space-independent assumption. Numerical evaluation is performed by the RTS (Reactor Transient Solution) code, written in FORTRAN II for the IBM-704 computer. The characteristic roots and residues which arise in this method of solution have been computed and are tabulated in detail for each of the main fissile species. Analytic or point-function reactivity variation may be introduced, together with constant or time-varying reactivity compensation, and the resulting power response, total energy release, and compensated reactivity computed precisely as functions of time. The code solves the general non-equilibrium kinetics problem with extraneous sources, the customary equilibrium solution being a special case of the general solution. Practical use of the method is demonstrated through computed response curves for representative reactivity-addition functions in various types of chain-reacting systems.

## GENERAL SOLUTION OF THE REACTOR KINETIC EQUATIONS

The general equations describing the time behavior of a chain reacting system are stated rigorously in terms of neutron transport theory. The usual reduction of the time-dependent transport equations to the more tractable one-velocity-group space-independent form (*1–3*) involves the introduction of an effective delayed neutron fraction, $\gamma\beta$, and an effective prompt neutron generation time, $\Lambda$. Both of these quantities are dependent on individual reactor geometry and, strictly speaking, both are functions of time, although the latter dependence can usually be ignored.

In the space-average approximation, the reactor kinetic equations may be written

$$\frac{dn(t)}{dt} = \frac{k(t)(1 - \bar{\gamma}\beta) - 1}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i \gamma_i C_i(t) + \gamma_s S(t) \tag{1a}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i k(t)}{\Lambda} n(t) - \lambda_i C_i(t) \qquad i = 1, 2 \cdots 6 \tag{1b}$$

where

$n(t) \equiv$ neutron density at time $t$.

$k(t) \equiv$ neutron reproduction number for finite geometry at time $t$.

$\Lambda \equiv$ prompt neutron generation time for finite geometry (*2*).

$\lambda_i =$ decay constant (reciprocal meanlife, $\tau_i^{-1}$) of the $i$th delayed neutron group.

$C_i(t) =$ density of $i$th delayed-neutron-group precursor at time $t$.

$\beta_i =$ $i$th group fraction of total neutrons from fission.

$\beta = \sum_i \beta_i =$ total delayed neutron fraction.

$\gamma_i =$ effectiveness (in producing fission) of $i$th group delayed neutrons compared to prompt neutrons.

$\bar{\gamma} \equiv \beta^{-1} \sum_i \gamma_i \beta_i =$ average delayed neutron effectiveness.

$\gamma_s S(t) \equiv$ effective contribution from extraneous sources at time $t$.

Integrating Eq. (1a)

$$C_i(t) = \frac{\beta_i}{\Lambda} \int_{-\infty}^{t} e^{-\lambda_i(t-t')} k(t') n(t')\, dt'$$

and substituting into (1a) yields

$$\frac{dn(t)}{dt} = \frac{k(t)(1 - \bar{\gamma}\beta) - 1}{\Lambda} + \int_{-\infty}^{t} \sum_{i=1}^{6} \frac{\lambda_i \gamma_i \beta_i}{\Lambda} e^{-\lambda_i(t-t')} k(t') n(t')\, dt' + \gamma_s S(t) \tag{2}$$

In principle, Eq. (2) could be solved numerically for $n(t)$ given the time dependence of $k(t)$ together

* Work performed under the auspices of the U. S. Atomic Energy Commission.

with appropriate initial conditions on $n$, $k$, $S$, and the $C_i$. However the appearance of prompt neutron generation time, $\Lambda$, as a factor in Eq. (2) imposes a severe limitation on the maximum time increment permissible in the computation, especially for "fast" systems where $\Lambda \sim 10^{-8}$ sec. An enormous number of points would have to be calculated, requiring prohibitively long computing time for all but the simplest case of constant $k$ (step function response) for which straightforward analytical solutions already exist. In the present development we shall simplify Eq. (2), keeping $k(t)$ arbitrary, and reduce it to a form convenient for numerical analysis.

We introduce $\delta k(t) \equiv k(t) - 1$ and assign the initial conditions: $n(0) \equiv$ neutron density at $t = 0$ and $C_i(0) \equiv$ precursor densities at $t = 0$ for $i = 1, 2, \cdots 6$. Then Eq. (2) can be rewritten

$$\frac{1 - \bar{\gamma}\beta}{\Lambda}\,\delta k(t)n(t) = \frac{dn(t)}{dt} + \frac{\bar{\gamma}\beta}{\Lambda}\,n(t) - \sum_{i=1}^{6} \frac{\gamma_i\beta_i\lambda_i}{\Lambda}\int_0^t e^{-\lambda_i(t-t')}n(t')\,dt' - \sum_{i=1}^{6} \frac{\gamma_i\beta_i\lambda_i}{\Lambda}\int_0^t e^{-\lambda_i(t-t')}\delta k(t')n(t')\,dt' - \sum_{i=1}^{6}\lambda_i\gamma_i C_i(0)e^{-\lambda_i t} - \gamma_s S(t)$$

Integrating the third term on the right-hand side by parts and combining similar terms

$$\begin{aligned}\frac{1 - \bar{\gamma}\beta}{\Lambda}\,\delta k(t)n(t) &= \frac{dn(t)}{dt} + \sum_{i=1}^{6} \frac{\gamma_i\beta_i}{\Lambda}\int_0^t e^{-\lambda_i(t-t')}\frac{dn(t')}{dt'}\,dt' \\ &\quad - \sum_{i=1}^{6} \frac{\gamma_i\beta_i\lambda_i}{\Lambda}\int_0^t e^{-\lambda_i(t-t')}\delta k(t')n(t')\,dt' \\ &\quad + \sum_{i=1}^{6}\gamma_i e^{-\lambda_i t}\left[\frac{\beta_i}{\Lambda}\,n(0) - \lambda_i C_i(0)\right] - \gamma_s S(t)\end{aligned} \tag{3}$$

Taking the Laplace transform, $\mathcal{L}$, of Eq. (3),[1] we have

$$\begin{aligned}&\left(1 + \mathcal{L}\left[\sum \frac{\gamma_i\beta_i}{\Lambda}\,e^{-\lambda_i t}\right]\right) s\left(\mathcal{L}[n(t)] - \frac{n(0)}{s}\right) \\ &= \left(\frac{1 - \bar{\gamma}\beta}{\Lambda} + \mathcal{L}\left[\sum \frac{\gamma_i\beta_i\lambda_i}{\Lambda}\,e^{-\lambda_i t}\right]\right)\mathcal{L}[\delta k(t)n(t)] \\ &\quad - \mathcal{L}\left[\sum \gamma_i e^{-\lambda_i t}\left(\frac{\beta_i n(0)}{\Lambda} - \lambda_i C_i(0)\right)\right] + \mathcal{L}[\gamma_s S(t)]\end{aligned} \tag{4}$$

Applying the inverse Laplacian, $\mathcal{L}^{-1}$, to Eq. (4) (again utilizing the convolution transformation) and solving for $n(t)$,

$$\begin{aligned}n(t) &= n(0) + \frac{1 - \bar{\gamma}\beta}{\Lambda}\int_0^t \mathcal{L}^{-1}[G(s)]\delta k(t')n(t')\,dt' \\ &\quad + \int_0^t \mathcal{L}^{-1}[H(s)]\delta k(t')n(t')\,dt' \\ &\quad + \int_0^t \left[\gamma_s S(t') - \sum_{i=1}^{6}\gamma_i e^{-\lambda_i t'}\left(\frac{\beta_i}{\Lambda}\,n(0) - \lambda_i C_i(0)\right)\right] \cdot \mathcal{L}^{-1}[G(s)]\,dt'\end{aligned} \tag{5}$$

with

$$G(s) \equiv s^{-1}(1 + \mathcal{L}\,[\sum \gamma_i\beta_i\Lambda^{-1}e^{-\lambda_i t}])^{-1}$$
$$H(s) \equiv G(s)(\mathcal{L}\,[\sum \gamma_i\beta_i\lambda_i\Lambda^{-1}e^{-\lambda_i t}])$$

In order to evaluate the inverse transforms of $G(s)$ and $H(s)$ we attempt expansions in partial fractions

$$G(s) = \left(s + \Lambda^{-1}\sum_{i=1}^{6}\frac{\gamma_i\beta_i s}{s + \lambda_i}\right)^{-1} = \sum_j \frac{B_j}{S - S_j} \tag{6}$$

$$H(s) = \frac{\sum_{i=1}^{6}\gamma_i\beta_i\lambda_i(s + \lambda_i)^{-1}}{s\Lambda + s\sum_{i=1}^{6}\gamma_i\beta_i(s + \lambda_i)^{-1}} = \sum_j \frac{R_j}{S - S_j{}^*} \tag{6a}$$

Expansions (6) and (6a) are possible in the present case since (1) the number of roots, $S_j$, is finite (it turns out $S_j = S_j{}^*$ for $j = 0, 1, \cdots 6$) and (2) all poles are simple (i.e., no essential singularities exist). Numerical values of the parameters $B_j$, $R_j$, and $S_j$ have been obtained with the IBM 704 digital computer[2] for various values of the prompt neutron generation time $\Lambda$, for each of the three main fissionable species, $U^{235}$, $Pu^{239}$, and $U^{233}$. The

[1] Here we employ standard Laplace transform methods (4) for solving Eq. (3). The integral terms in Eq. (3) are directly transformed by the convolution (Faltung) theorem; thus for the first integral term:

$$\mathcal{L}\left[\sum \frac{\gamma_i\beta_i}{\Lambda}\int_0^t e^{-\lambda_i(t-t')}\frac{dn(t')}{dt'}\,dt'\right] = \mathcal{L}\left[\sum \frac{\gamma_i\beta_i}{\Lambda}\,e^{-\lambda_i t}\right]\mathcal{L}\left[\frac{dn(t)}{dt}\right] = \sum_{i=1}^{6}\frac{\gamma_i\beta_i}{\Lambda(s + \lambda_i)}\,(s\mathcal{L}[n(t)] - n(0))$$

where $s$ is the transform variable.

[2] In the computation $G(s)$ is first expressed as a ratio of two polynomials: $G(s) = P(s)/Q(s)$. Then $B_j = P(S_j)/Q'(S_j)$ where the $S_j$ are the seven roots of $Q(s) = 0$. The same procedure is used to obtain $R_j$. Roots of $Q(s) = 0$ were extracted by LASL code S-871, a general polynomial solver subroutine; $B_j$ and $R_j$ were evaluated using the RTR code, written specifically for this problem.

most recent delayed neutron data (*5, 6*) were used in these calculations and we have taken $\gamma_i = \bar{\gamma} =$ unity.[3] Computed values of $S_j$, $B_j$, and $A_j \equiv [(1-\beta)/\Lambda]B_j + R_j$ are tabulated in Tables I–III, which were lithographed directly from the IBM 704 print-out format. By definition, the $S_j$ and $A_j$ have units of reciprocal seconds, while the $B_j$ are dimensionless.

Having obtained the roots and coefficients of Eq. (6), the inverse transforms of $G(s)$ and $H(s)$ (evaluated at time $t - t'$) are obtained directly through term by term inversion of Eqs. (6) and (6a); thus

$$\mathcal{L}^{-1}[G(s)]\,|_{t-t'} = \mathcal{L}^{-1}\left[\sum_{j=0}^{6} \frac{B_j}{S - S_j}\right] = \sum_{j=0}^{6} B_j e^{S_j(t-t')}$$

$$\mathcal{L}^{-1}[H(s)]\,|_{t-t'} = \mathcal{L}^{-1}\left[\sum_{j=0}^{6} \frac{R_j}{S - S_j^*}\right] = \sum_{j=0}^{6} R_j e^{S_j^*(t-t')}$$

Recalling that $S_j = S_j^*$ identically and $A_j \equiv [(1-\beta)/\Lambda]B_j + R_j$, Eq. (5) reduces to

$$n(t) = n(0) + \sum_{j=0}^{6} A_j \int_0^t e^{S_j(t-t')}\delta k(t')n(t')\,dt' + \Omega_0(t) \tag{7}$$

with $\Omega_0(t)$ defined as

$$\Omega_0(t) \equiv \int_0^t \left[\gamma_s S(t') + \sum_{i=1}^{6} (\lambda_i C_i(0) - \Lambda^{-1}\beta_i n(0)) \cdot \gamma_i e^{-\lambda_i t'}\right] \sum_{j=0}^{6} B_j e^{S_j(t-t')}\,dt'$$

Equation (7) represents a general solution of the reactor kinetic equations for arbitrary $\delta k(t)$, in a form readily adaptable to numerical solution by high-speed digital computers. The quantity $\Omega_0(t)$, can be calculated for specified values of $n(0)$, $C_i(0)$, and $\gamma_s S(t)$, together with appropriate values of $S_j$ and $B_j$ from Tables I–III. Nonequilibrium kinetics solutions (i.e., for $\Omega_0(t) \neq 0$) have been obtained for several representative problems. In most cases it may be assumed that $\gamma_i = \bar{\gamma} =$ unity, and that extraneous source contributions are negligible.[4] For the most common case of equilibrium (delayed critical operation prior to $t = 0$), $\dot{C}_i(0) = 0$, and $k(0) = 1$. Under these conditions, Eq. (1a) reduces to

$$\frac{\beta_i n(0)}{\Lambda} = \lambda_i C_i(0) \qquad i = 1, 2 \cdots 6$$

and $\Omega_0(t)$ vanishes for the source-free case.

It may be noted that $A_j$, $S_j$, and $B_j$ in Tables I–III are given to five figures while individual delayed neutron parameters (*5, 6*) are only accurate to within a few per cent. As explained in reference *5*, the errors indicated for individual delayed neutron parameters were evaluated as part of the iterative-least-squares fit computation. In so far as reactor kinetic behavior is concerned, these errors are effectively an order of magnitude smaller, as demonstrated by the comparison of six-group inhour relations with their corresponding Laplace-transformed period-reactivity relations (*8*).

It is noteworthy that in Eq. (7) [and its numerical counterpart, Eq. (7a), below] we have formal separation of the time dependent $\delta k(t)n(t)$ product from the characteristic parameters $A_j$, $B_j$, and $S_j$, which may be considered universal constants of a particular fissile species.[5] This may be contrasted with the analogous roots of the simple inhour equation which are directly dependent on $k$ (*6*). In the numerical solution of Eq. (7), neutron density is given explicitly as a function of time, no iteration being required as is the case with many numerical methods for solution of the reactor kinetic equations. For many kinetics problems this simplification represents a considerable saving in computing time, and in general operational convenience.

The explicit numerical solution of Eq. (7) for neutron density, $n_m$, in the $m$th time interval may be written

$$n_m \cong \frac{n(0) + \sum_{j=0}^{6} A_j e^{S_j mh} \sum_{l=0}^{m-1} e^{-S_j lh}\delta k_l\, n_l\, h + \Omega_{0_m}}{1 - \sum_{j=0}^{6} A_j\, \delta k_m\, h} \tag{7a}$$

where $h$, the integration time interval, is restricted to values less than

$$\left[\sum_{j=0}^{6} A_j\, \delta k_m\right]^{-1} \simeq \Lambda/\delta k_m$$

in order to obtain finite $n_m$. Recursion relations

[3] Taking $\gamma_i = \bar{\gamma} =$ unity is required for the purpose of generalization, and should be applicable in most practical cases. In specific instances where individual $\gamma_i$ and/or $\bar{\gamma}$ are known, and are appreciably different from unity, the appropriate constants $S_j$, $A_j$, and $B_j$ may be computed (for given $\Lambda$) using the RTR code and S-871 subroutine.

[4] At very low flux levels, as in reactor startup, the source perturbation on kinetic behavior can be appreciable (*7*). However, many reactor control problems are concerned with power levels at which source perturbation is negligible, and the source-free form of Eq. (7) may be safely assumed.

[5] In the most general interpretation, $S_j$ are actually the seven roots (stable plus six transient periods) of the inhour equation for any arbitrary value of $k$, which must by definition be a constant; i.e., $\delta k \equiv 0$. If we generalize the definition of excess reactivity, $\delta k \equiv k - k_0$, the $S_j$ would then correspond to the roots of the inhour equation at $k = k_0$ (cf., plot of inhour-equation roots versus $k$ in Fig. 2, reference 6. Following customary practice, we have taken $k_0$ equal to unity; the $S_j$ are then simply the seven roots of the inhour equation at delayed critical ($k_0 \equiv 1$),

TABLE I-A
CHARACTERISTIC ROOTS, $S_j$, FOR $U^{235}$ [a]

| Λ | $-S_0$ | $-S_1$ | $-S_2$ | $-S_3$ | $-S_4$ | $-S_5$ | $-S_6$ |
|---|---|---|---|---|---|---|---|
| 1.−08 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 6.5000 05 |
| 2.−08 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 3.2500 05 |
| 3.−08 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 2.1666 05 |
| 4.−08 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 1.6250 05 |
| 5.−08 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 1.3000 05 |
| 6.−08 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 1.0833 05 |
| 7.−08 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 9.2857 04 |
| 8.−08 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 8.1250 04 |
| 9.−08 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 7.2222 04 |
| 1.−07 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 6.5000 04 |
| 2.−07 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 3.2500 04 |
| 3.−07 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 2.1667 04 |
| 4.−07 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 1.6250 04 |
| 5.−07 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 1.3000 04 |
| 6.−07 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7801 00 | 1.0833 04 |
| 7.−07 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7800 00 | 9.2861 03 |
| 8.−07 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7800 00 | 8.1254 03 |
| 9.−07 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7800 00 | 7.2226 03 |
| 1.−06 | 0.0000 00 | 1.5052−02 | 7.0999−02 | 1.9852−01 | 1.2404 00 | 3.7800 00 | 6.5004 03 |
| 2.−06 | 0.0000 00 | 1.5052−02 | 7.0998−02 | 1.9851−01 | 1.2403 00 | 3.7800 00 | 3.2504 03 |
| 3.−06 | 0.0000 00 | 1.5052−02 | 7.0997−02 | 1.9851−01 | 1.2403 00 | 3.7799 00 | 2.1671 03 |
| 4.−06 | 0.0000 00 | 1.5052−02 | 7.0997−02 | 1.9851−01 | 1.2403 00 | 3.7799 00 | 1.6254 03 |
| 5.−06 | 0.0000 00 | 1.5052−02 | 7.0996−02 | 1.9851−01 | 1.2403 00 | 3.7798 00 | 1.3004 03 |
| 6.−06 | 0.0000 00 | 1.5052−02 | 7.0996−02 | 1.9851−01 | 1.2402 00 | 3.7798 00 | 1.0837 03 |
| 7.−06 | 0.0000 00 | 1.5052−02 | 7.0995−02 | 1.9850−01 | 1.2402 00 | 3.7798 00 | 9.2900 02 |
| 8.−06 | 0.0000 00 | 1.5052−02 | 7.0994−02 | 1.9850−01 | 1.2402 00 | 3.7797 00 | 8.1293 02 |
| 9.−06 | 0.0000 00 | 1.5052−02 | 7.0994−02 | 1.9850−01 | 1.2402 00 | 3.7797 00 | 7.2265 02 |
| 1.−05 | 0.0000 00 | 1.5052−02 | 7.0993−02 | 1.9850−01 | 1.2401 00 | 3.7796 00 | 6.5043 02 |
| 2.−05 | 0.0000 00 | 1.5052−02 | 7.0987−02 | 1.9847−01 | 1.2399 00 | 3.7791 00 | 3.2543 02 |
| 3.−05 | 0.0000 00 | 1.5052−02 | 7.0981−02 | 1.9845−01 | 1.2396 00 | 3.7787 00 | 2.1710 02 |
| 4.−05 | 0.0000 00 | 1.5052−02 | 7.0975−02 | 1.9843−01 | 1.2393 00 | 3.7782 00 | 1.6293 02 |
| 5.−05 | 0.0000 00 | 1.5051−02 | 7.0969−02 | 1.9841−01 | 1.2391 00 | 3.7777 00 | 1.3043 02 |
| 6.−05 | 0.0000 00 | 1.5051−02 | 7.0963−02 | 1.9838−01 | 1.2388 00 | 3.7772 00 | 1.0877 02 |
| 7.−05 | 0.0000 00 | 1.5051−02 | 7.0957−02 | 1.9836−01 | 1.2385 00 | 3.7767 00 | 9.3297 01 |
| 8.−05 | 0.0000 00 | 1.5051−02 | 7.0951−02 | 1.9834−01 | 1.2383 00 | 3.7762 00 | 8.1691 01 |
| 9.−05 | 0.0000 00 | 1.5051−02 | 7.0945−02 | 1.9831−01 | 1.2380 00 | 3.7757 00 | 7.2664 01 |
| 1.−04 | 0.0000 00 | 1.5050−02 | 7.0939−02 | 1.9829−01 | 1.2377 00 | 3.7752 00 | 6.5443 01 |
| 2.−04 | 0.0000 00 | 1.5048−02 | 7.0878−02 | 1.9807−01 | 1.2350 00 | 3.7697 00 | 3.2951 01 |
| 3.−04 | 0.0000 00 | 1.5046−02 | 7.0817−02 | 1.9784−01 | 1.2321 00 | 3.7637 00 | 2.2127 01 |
| 4.−04 | 0.0000 00 | 1.5044−02 | 7.0757−02 | 1.9761−01 | 1.2292 00 | 3.7568 00 | 1.6720 01 |
| 5.−04 | 0.0000 00 | 1.5042−02 | 7.0696−02 | 1.9739−01 | 1.2262 00 | 3.7491 00 | 1.3481 01 |
| 6.−04 | 0.0000 00 | 1.5040−02 | 7.0636−02 | 1.9716−01 | 1.2231 00 | 3.7403 00 | 1.1327 01 |
| 7.−04 | 0.0000 00 | 1.5038−02 | 7.0575−02 | 1.9694−01 | 1.2199 00 | 3.7302 00 | 9.7932 00 |
| 8.−04 | 0.0000 00 | 1.5036−02 | 7.0515−02 | 1.9671−01 | 1.2167 00 | 3.7186 00 | 8.6477 00 |
| 9.−04 | 0.0000 00 | 1.5034−02 | 7.0454−02 | 1.9649−01 | 1.2133 00 | 3.7050 00 | 7.7622 00 |
| 1.−03 | 0.0000 00 | 1.5032−02 | 7.0394−02 | 1.9626−01 | 1.2099 00 | 3.6891 00 | 7.0596 00 |

[a] Universal constants, $S_j$, in Eq. (7), calculated using $U^{235}$ delayed neutron data of reference 6. Parameter Λ is prompt neutron lifetime. Floating decimal notation: 1.2345 03 ≡ 1234.5; 5.−08 ≡ 5 × $10^{-8}$, etc *All roots are negative: note that $-S_j$ values are tabulated.*

have been developed which greatly reduce the number of numerical operations formally indicated by Eq. (7a). The general numerical solution for $n(t)$ has been coded in FORTRAN II, and is designated the RTS (Reactor Transient Solution) code; cf., Appendix A.

The indicated summation over $l$ in Eq. (7a) [corresponding to integration over $t'$ in Eq. (7)] may

TABLE I-B

CHARACTERISTIC COEFFICIENTS, $A_j$, FOR $U^{235}$[a]

| Λ | $A_0$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| 1.−08 | 1.2063 01 | 1.3349 00 | 8.5532 00 | 1.1402 01 | 2.1302 01 | 1.2305 01 | 9.9349 07 |
| 2.−08 | 1.2063 01 | 1.3349 00 | 8.5532 00 | 1.1402 01 | 2.1302 01 | 1.2305 01 | 4.9674 07 |
| 3.−08 | 1.2063 01 | 1.3349 00 | 8.5532 00 | 1.1402 01 | 2.1302 01 | 1.2305 01 | 3.3116 07 |
| 4.−08 | 1.2063 01 | 1.3349 00 | 8.5532 00 | 1.1402 01 | 2.1302 01 | 1.2306 01 | 2.4837 07 |
| 5.−08 | 1.2063 01 | 1.3349 00 | 8.5532 00 | 1.1402 01 | 2.1302 01 | 1.2306 01 | 1.9869 07 |
| 6.−08 | 1.2063 01 | 1.3349 00 | 8.5532 00 | 1.1402 01 | 2.1302 01 | 1.2306 01 | 1.6558 07 |
| 7.−08 | 1.2063 01 | 1.3349 00 | 8.5532 00 | 1.1402 01 | 2.1302 01 | 1.2306 01 | 1.4192 07 |
| 8.−08 | 1.2063 01 | 1.3349 00 | 8.5532 00 | 1.1502 01 | 2.1302 01 | 1.2306 01 | 1.2418 07 |
| 9.−08 | 1.2063 01 | 1.3349 00 | 8.5532 00 | 1.1402 01 | 2.1302 01 | 1.2306 01 | 1.1038 07 |
| 1.−07 | 1.2063 01 | 1.3349 00 | 8.5532 00 | 1.1402 01 | 2.1303 01 | 1.2306 01 | 9.9349 06 |
| 2.−07 | 1.2063 01 | 1.3349 00 | 8.5532 00 | 1.1402 01 | 2.1303 01 | 1.2308 01 | 4.9674 06 |
| 3.−07 | 1.2063 01 | 1.3349 00 | 8.5532 00 | 1.1402 01 | 2.1304 01 | 1.2309 01 | 3.3115 06 |
| 4.−07 | 1.2063 01 | 1.3349 00 | 8.5532 00 | 1.1402 01 | 2.1304 01 | 1.2310 01 | 2.4836 06 |
| 5.−07 | 1.2063 01 | 1.3348 00 | 8.5532 00 | 1.1402 01 | 2.1305 01 | 1.2311 01 | 1.9869 06 |
| 6.−07 | 1.2063 01 | 1.3348 00 | 8.5532 00 | 1.1402 01 | 2.1306 01 | 1.2313 01 | 1.6557 06 |
| 7.−07 | 1.2063 01 | 1.3348 00 | 8.5532 00 | 1.1402 01 | 2.1306 01 | 1.2314 01 | 1.4192 06 |
| 8.−07 | 1.2063 01 | 1.3348 00 | 8.5532 00 | 1.1402 01 | 2.1307 01 | 1.2315 01 | 1.2418 06 |
| 9.−07 | 1.2063 01 | 1.3348 00 | 8.5532 00 | 1.1402 01 | 2.1308 01 | 1.2316 01 | 1.1038 06 |
| 1.−06 | 1.2063 01 | 1.3348 00 | 8.5532 00 | 1.1402 01 | 2.1308 01 | 1.2318 01 | 9.9343 05 |
| 2.−06 | 1.2063 01 | 1.3348 00 | 8.5532 00 | 1.1402 01 | 2.1315 01 | 1.2330 01 | 4.9668 05 |
| 3.−06 | 1.2063 01 | 1.3348 00 | 8.5532 00 | 1.1402 01 | 2.1321 01 | 1.2343 01 | 3.3109 05 |
| 4.−06 | 1.2062 01 | 1.3348 00 | 8.5532 00 | 1.1402 01 | 2.1327 01 | 1.2355 01 | 2.4830 05 |
| 5.−06 | 1.2062 01 | 1.3348 00 | 8.5532 00 | 1.1402 01 | 2.1334 01 | 1.2368 01 | 1.9863 05 |
| 6.−06 | 1.2062 01 | 1.3347 00 | 8.5532 00 | 1.1402 01 | 2.1340 01 | 1.2380 01 | 1.6551 05 |
| 7.−06 | 1.2062 01 | 1.3347 00 | 8.5533 00 | 1.1402 01 | 2.1347 01 | 1.2393 01 | 1.4186 05 |
| 8.−06 | 1.2062 01 | 1.3347 00 | 8.5533 00 | 1.1402 01 | 2.1353 01 | 1.2406 01 | 1.2412 05 |
| 9.−06 | 1.2062 01 | 1.3347 00 | 8.5533 00 | 1.1402 01 | 2.1359 01 | 1.2418 01 | 1.1032 05 |
| 1.−05 | 1.2062 01 | 1.3347 00 | 8.5533 00 | 1.1402 01 | 2.1366 01 | 1.2431 01 | 9.9282 04 |
| 2.−05 | 1.2060 01 | 1.3345 00 | 8.5534 00 | 1.1403 01 | 2.1430 01 | 1.2559 01 | 4.9607 04 |
| 3.−05 | 1.2059 01 | 1.3343 00 | 8.5535 00 | 1.1404 01 | 2.1494 01 | 1.2688 01 | 3.3049 04 |
| 4.−05 | 1.2057 01 | 1.3341 00 | 8.5537 00 | 1.1404 01 | 2.1559 01 | 1.2820 01 | 2.4769 04 |
| 5.−05 | 1.2056 01 | 1.3339 00 | 8.5538 00 | 1.1405 01 | 2.1623 01 | 1.2954 01 | 1.9802 04 |
| 6.−05 | 1.2054 01 | 1.3338 00 | 8.5539 00 | 1.1405 01 | 2.1688 01 | 1.3089 01 | 1.6490 04 |
| 7.−05 | 1.2053 01 | 1.3336 00 | 8.5540 00 | 1.1406 01 | 2.1754 01 | 1.3227 01 | 1.4124 04 |
| 8.−05 | 1.2051 01 | 1.3334 00 | 8.5541 00 | 1.1406 01 | 2.1819 01 | 1.3367 01 | 1.2350 04 |
| 9.−05 | 1.2050 01 | 1.3332 00 | 8.5542 00 | 1.1407 01 | 2.1885 01 | 1.3509 01 | 1.0970 04 |
| 1.−04 | 1.2049 01 | 1.3330 00 | 8.5544 00 | 1.1408 01 | 2.1951 01 | 1.3653 01 | 9.8660 03 |
| 2.−04 | 1.2034 01 | 1.3312 00 | 8.5555 00 | 1.1413 01 | 2.2623 01 | 1.5228 01 | 4.8963 03 |
| 3.−04 | 1.2020 01 | 1.3294 00 | 8.5565 00 | 1.1418 01 | 2.3321 01 | 1.7080 01 | 3.2379 03 |
| 4.−04 | 1.2005 01 | 1.3275 00 | 8.5574 00 | 1.1423 01 | 2.4043 01 | 1.9276 01 | 2.4071 03 |
| 5.−04 | 1.1991 01 | 1.3257 00 | 8.5582 00 | 1.1428 01 | 2.4792 01 | 2.1900 01 | 1.9070 03 |
| 6.−04 | 1.1976 01 | 1.3239 00 | 8.5589 00 | 1.1432 01 | 2.5568 01 | 2.5062 01 | 1.5719 03 |
| 7.−04 | 1.1962 01 | 1.3221 00 | 8.5595 00 | 1.1436 01 | 2.6371 01 | 2.8909 01 | 1.3307 03 |
| 8.−04 | 1.1948 01 | 1.3203 00 | 8.5601 00 | 1.1440 01 | 2.7203 01 | 3.3632 01 | 1.1477 03 |
| 9.−04 | 1.1933 01 | 1.3185 00 | 8.5605 00 | 1.1443 01 | 2.8063 01 | 3.9481 01 | 1.0030 03 |
| 1.−03 | 1.1919 01 | 1.3167 00 | 8.5608 00 | 1.1446 01 | 2.8952 01 | 4.6785 01 | 8.8451 02 |

[a] Universal constants, $A_j$, in Eq. (7), calculated using $U^{235}$ delayed neutron data of reference *6*. Parameter Λ is prompt neutron lifetime. Floating decimal notation: 1,2345 03 ≡ 1234.5; 5.−08 ≡ $5 \times 10^{-8}$; etc.

be evaluated using either the trapezoidal rule or Simpson's rule for numerical integration. The implicit assumption that the integrand can be accurately represented by a linear (or quadratic) expression over the appropriate time interval(s) is clearly valid as long as $h$ is kept sufficiently small. Subject to this limitation, however, it is desirable to keep $h$ reasonably large to reduce the number of

TABLE I-C

CHARACTERISTIC COEFFICIENTS, $B_j$, FOR $U^{235}$[a]

| Λ | $B_0$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ |
|---|---|---|---|---|---|---|---|
| 1.−08 | 1.2063−07 | 1.3349−08 | 8.5532−08 | 1.1402−07 | 2.1302−07 | 1.2305−07 | 9.9999−01 |
| 2.−08 | 2.4127−07 | 2.6698−08 | 1.7106−07 | 2.2804−07 | 4.2604−07 | 2.4611−07 | 9.9999−01 |
| 3.−08 | 3.6190−07 | 4.0047−08 | 2.5659−07 | 3.4206−07 | 6.3907−07 | 3.6917−07 | 9.9999−01 |
| 4.−08 | 4.8254−07 | 5.3396−08 | 3.4212−07 | 4.5608−07 | 8.5210−07 | 4.9224−07 | 9.9999−01 |
| 5.−08 | 6.0317−07 | 6.6745−08 | 4.2766−07 | 5.7011−07 | 1.0651−06 | 6.1531−07 | 9.9999−01 |
| 6.−08 | 7.2381−07 | 8.0094−08 | 5.1319−07 | 6.8413−07 | 1.2781−06 | 7.3837−07 | 9.9999−01 |
| 7.−08 | 8.4444−07 | 9.3443−08 | 5.9872−07 | 7.9815−07 | 1.4911−06 | 8.6144−07 | 9.9999−01 |
| 8.−08 | 9.6508−07 | 1.0679−07 | 6.8425−07 | 9.1217−07 | 1.7042−06 | 9.8452−07 | 9.9999−01 |
| 9.−08 | 1.0857−06 | 1.2014−07 | 7.6978−07 | 1.0261−06 | 1.9172−06 | 1.1076−06 | 9.9999−01 |
| 1.−07 | 1.2063−06 | 1.3349−07 | 8.5532−07 | 1.1402−06 | 2.1303−06 | 1.2306−06 | 9.9999−01 |
| 2.−07 | 2.4127−06 | 2.6698−07 | 1.7106−06 | 2.2804−06 | 4.2607−06 | 2.4616−06 | 9.9998−01 |
| 3.−07 | 3.6190−06 | 4.0047−07 | 2.5659−06 | 3.4206−06 | 6.3912−06 | 3.6927−06 | 9.9997−01 |
| 4.−07 | 4.8253−06 | 5.3396−07 | 3.4212−06 | 4.5608−06 | 8.5219−06 | 4.9242−06 | 9.9997−01 |
| 5.−07 | 6.0317−06 | 6.6744−07 | 4.2766−06 | 5.7011−06 | 1.0652−05 | 6.1559−06 | 9.9996−01 |
| 6.−07 | 7.2380−06 | 8.0093−07 | 5.1319−06 | 6.8413−06 | 1.2783−05 | 7.3878−06 | 9.9995−01 |
| 7.−07 | 8.4444−06 | 9.3442−07 | 5.9872−06 | 7.9815−06 | 1.4914−05 | 8.6200−06 | 9.9995−01 |
| 8.−07 | 9.6507−06 | 1.0679−06 | 6.8425−06 | 9.1218−06 | 1.7045−05 | 9.8524−06 | 9.9994−01 |
| 9.−07 | 1.0857−05 | 1.2014−06 | 7.6979−06 | 1.0262−05 | 1.9177−05 | 1.1085−05 | 9.9993−01 |
| 1.−06 | 1.2063−05 | 1.3348−06 | 8.5532−06 | 1.1402−05 | 2.1308−05 | 1.2318−05 | 9.9993−01 |
| 2.−06 | 2.4126−05 | 2.6697−06 | 1.7106−05 | 2.2804−05 | 4.2630−05 | 2.4661−05 | 9.9986−01 |
| 3.−06 | 3.6189−05 | 4.0045−06 | 2.5659−05 | 3.4207−05 | 6.3964−05 | 3.7029−05 | 9.9979−01 |
| 4.−06 | 4.8251−05 | 5.3393−06 | 3.4213−05 | 4.5609−05 | 8.5311−05 | 4.9423−05 | 9.9973−01 |
| 5.−06 | 6.0314−05 | 6.6740−06 | 4.2766−05 | 5.7012−05 | 1.0667−04 | 6.1842−05 | 9.9966−01 |
| 6.−06 | 7.2376−05 | 8.0087−06 | 5.1319−05 | 6.8415−05 | 1.2804−04 | 7.4286−05 | 9.9959−01 |
| 7.−06 | 8.4437−05 | 9.3434−06 | 5.9873−05 | 7.9818−05 | 1.4943−04 | 8.6756−05 | 9.9953−01 |
| 8.−06 | 9.6499−05 | 1.0678−05 | 6.8426−05 | 9.1221−05 | 1.7082−04 | 9.9251−05 | 9.9946−01 |
| 9.−06 | 1.0856−04 | 1.2012−05 | 7.6979−05 | 1.0262−04 | 1.9224−04 | 1.1177−04 | 9.9939−01 |
| 1.−05 | 1.2062−04 | 1.3347−05 | 8.5533−05 | 1.1402−04 | 2.1366−04 | 1.2431−04 | 9.9932−01 |
| 2.−05 | 2.4121−04 | 2.6690−05 | 1.7106−04 | 2.2806−04 | 4.2861−04 | 2.5120−04 | 9.9865−01 |
| 3.−05 | 3.6177−04 | 4.0030−05 | 2.5660−04 | 3.4212−04 | 6.4485−04 | 3.8070−04 | 9.9797−01 |
| 4.−05 | 4.8230−04 | 5.3367−05 | 3.4214−04 | 4.5618−04 | 8.6240−04 | 5.1289−04 | 9.9729−01 |
| 5.−05 | 6.0281−04 | 6.6699−05 | 4.2769−04 | 5.7026−04 | 1.0812−03 | 6.4783−04 | 9.9660−01 |
| 6.−05 | 7.2328−04 | 8.0028−05 | 5.1323−04 | 6.8435−04 | 1.3014−03 | 7.8557−04 | 9.9591−01 |
| 7.−05 | 8.4373−04 | 9.3353−05 | 5.9878−04 | 7.9845−04 | 1.5229−03 | 9.2618−04 | 9.9521−01 |
| 8.−05 | 9.6415−04 | 1.0667−04 | 6.8433−04 | 9.1257−04 | 1.7457−03 | 1.0697−03 | 9.9451−01 |
| 9.−05 | 1.0845−03 | 1.1999−04 | 7.6989−04 | 1.0266−03 | 1.9698−03 | 1.2162−03 | 9.9381−01 |
| 1.−04 | 1.2049−03 | 1.3330−04 | 8.5544−04 | 1.1408−03 | 2.1953−03 | 1.3659−03 | 9.9310−01 |
| 2.−04 | 2.4069−03 | 2.6624−04 | 1.7111−03 | 2.2828−03 | 4.5258−03 | 3.0480−03 | 9.8575−01 |
| 3.−04 | 3.6060−03 | 3.9882−04 | 2.5670−03 | 3.4258−03 | 6.9989−03 | 5.1300−03 | 9.7787−01 |
| 4.−04 | 4.8022−03 | 5.3103−04 | 3.4230−03 | 4.5699−03 | 9.6222−03 | 7.7221−03 | 9.6932−01 |
| 5.−04 | 5.9956−03 | 6.6288−04 | 4.2792−03 | 5.7147−03 | 1.2404−02 | 1.0970−02 | 9.5997−01 |
| 6.−04 | 7.1861−03 | 7.9437−04 | 5.1356−03 | 6.8604−03 | 1.5352−02 | 1.5071−02 | 9.4959−01 |
| 7.−04 | 8.3737−03 | 9.2550−04 | 5.9920−03 | 8.0067−03 | 1.8476−02 | 2.0289−02 | 9.3793−01 |
| 8.−04 | 9.5585−03 | 1.0562−03 | 6.8484−03 | 9.1536−03 | 2.1783−02 | 2.6986−02 | 9.2461−01 |
| 9.−04 | 1.0740−02 | 1.1866−03 | 7.7049−03 | 1.0300−02 | 2.5284−02 | 3.5652−02 | 9.0913−01 |
| 1.−03 | 1.1919−02 | 1.3167−03 | 8.5614−03 | 1.1448−02 | 2.8987−02 | 4.6959−02 | 8.9080−01 |

[a] Universal constants, $B_j$, in Eq. 7, calculated using $U^{235}$ delayed neutron data of reference 6. Parameter Λ is prompt neutron lifetime. Floating decimal notation: 1.2345 03 ≡ 1234.5; 5. − 08 ≡ 5 × $10^{-8}$, etc.

computed points, minimize possible round-off error, etc. For some problems a single integration interval ($h$) can be used over the entire time range of interest. In general, however, it is desirable to vary $h$ automatically as dictated by functional behavior of the problem. Thus ideally the time scale is expanded or contracted to provide near-optimum time intervals for local $\delta k(t)$ and $n(t)$ variation. In the RTS

TABLE II-A

CHARACTERISTIC ROOTS, $S_j$, FOR $Pu^{239}$[a]

| $\Lambda$ | $-S_0$ | $-S_1$ | $-S_2$ | $-S_3$ | $-S_4$ | $-S_5$ | $-S_6$ |
|---|---|---|---|---|---|---|---|
| 1.−08 | 0.0000 00 | 1.4789−02 | 8.4405−02 | 2.3738−01 | 1.1437 00 | 3.1098 00 | 2.1000 05 |
| 2.−08 | 0.0000 00 | 1.4789−02 | 8.4405−02 | 2.3738−01 | 1.1437 00 | 3.1098 00 | 1.0500 05 |
| 3.−08 | 0.0000 00 | 1.4789−02 | 8.4405−02 | 2.3738−01 | 1.1437 00 | 3.1098 00 | 7.0000 04 |
| 4.−08 | 0.0000 00 | 1.4789−02 | 8.4404−02 | 2.3737−01 | 1.1437 00 | 3.1098 00 | 5.2500 04 |
| 5.−08 | 0.0000 00 | 1.4789−02 | 8.4404−02 | 2.3737−01 | 1.1437 00 | 3.1098 00 | 4.2000 04 |
| 6.−08 | 0.0000 00 | 1.4789−02 | 8.4404−02 | 2.3737−01 | 1.1437 00 | 3.1098 00 | 3.5000 04 |
| 7.−08 | 0.0000 00 | 1.4789−02 | 8.4404−02 | 2.3737−01 | 1.1437 00 | 3.1098 00 | 3.0000 04 |
| 8.−08 | 0.0000 00 | 1.4789−02 | 8.4404−02 | 2.3737−01 | 1.1437 00 | 3.1098 00 | 2.6250 04 |
| 9.−08 | 0.0000 00 | 1.4789−02 | 8.4404−02 | 2.3737−01 | 1.1437 00 | 3.1098 00 | 2.3333 04 |
| 1.−07 | 0.0000 00 | 1.4789−02 | 8.4404−02 | 2.3737−01 | 1.1437 00 | 3.1098 00 | 2.1000 04 |
| 2.−07 | 0.0000 00 | 1.4789−02 | 8.4404−02 | 2.3737−01 | 1.1437 00 | 3.1098 00 | 1.0500 04 |
| 3.−07 | 0.0000 00 | 1.4789−02 | 8.4404−02 | 2.3737−01 | 1.1437 00 | 3.1098 00 | 7.0003 03 |
| 4.−07 | 0.0000 00 | 1.4789−02 | 8.4404−02 | 2.3737−01 | 1.1437 00 | 3.1098 00 | 5.2503 03 |
| 5.−07 | 0.0000 00 | 1.4789−02 | 8.4403−02 | 2.3737−01 | 1.1436 00 | 3.1097 00 | 4.2003 03 |
| 6.−07 | 0.0000 00 | 1.4789−02 | 8.4403−02 | 2.3737−01 | 1.1436 00 | 3.1097 00 | 3.5003 03 |
| 7.−07 | 0.0000 00 | 1.4789−02 | 8.4403−02 | 2.3737−01 | 1.1436 00 | 3.1097 00 | 3.0003 03 |
| 8.−07 | 0.0000 00 | 1.4789−02 | 8.4403−02 | 2.3737−01 | 1.1436 00 | 3.1097 00 | 2.6253 03 |
| 9.−07 | 0.0000 00 | 1.4789−02 | 8.4402−02 | 2.3737−01 | 1.1436 00 | 3.1097 00 | 2.3337 03 |
| 1.−06 | 0.0000 00 | 1.4789−02 | 8.4402−02 | 2.3737−01 | 1.1436 00 | 3.1097 00 | 2.1003 03 |
| 2.−06 | 0.0000 00 | 1.4789−02 | 8.4400−02 | 2.3736−01 | 1.1436 00 | 3.1095 00 | 1.0503 03 |
| 3.−06 | 0.0000 00 | 1.4789−02 | 8.4397−02 | 2.3735−01 | 1.1435 00 | 3.1094 00 | 7.0038 02 |
| 4.−06 | 0.0000 00 | 1.4789−02 | 8.4395−02 | 2.3735−01 | 1.1435 00 | 3.1093 00 | 5.2538 02 |
| 5.−06 | 0.0000 00 | 1.4789−02 | 8.4393−02 | 2.3734−01 | 1.1434 00 | 3.1091 00 | 4.2038 02 |
| 6.−06 | 0.0000 00 | 1.4789−02 | 8.4390−02 | 2.3733−01 | 1.1433 00 | 3.1090 00 | 3.5039 02 |
| 7.−06 | 0.0000 00 | 1.4789−02 | 8.4388−02 | 2.3732−01 | 1.1433 00 | 3.1089 00 | 3.0039 02 |
| 8.−06 | 0.0000 00 | 1.4789−02 | 8.4385−02 | 2.3732−01 | 1.1432 00 | 3.1087 00 | 2.6289 02 |
| 9.−06 | 0.0000 00 | 1.4789−02 | 8.4383−02 | 2.3731−01 | 1.1432 00 | 3.1086 00 | 2.3372 02 |
| 1.−05 | 0.0000 00 | 1.4789−02 | 8.4381−02 | 2.3730−01 | 1.1431 00 | 3.1085 00 | 2.1039 02 |
| 2.−05 | 0.0000 00 | 1.4788−02 | 8.4357−02 | 2.3723−01 | 1.1426 00 | 3.1071 00 | 1.0539 02 |
| 3.−05 | 0.0000 00 | 1.4788−02 | 8.4333−02 | 2.3716−01 | 1.1420 00 | 3.1057 00 | 7.0394 01 |
| 4.−05 | 0.0000 00 | 1.4787−02 | 8.4309−02 | 2.3708−01 | 1.1414 00 | 3.1043 00 | 5.2897 01 |
| 5.−05 | 0.0000 00 | 1.4787−02 | 8.4285−02 | 2.3701−01 | 1.1408 00 | 3.1028 00 | 4.2399 01 |
| 6.−05 | 0.0000 00 | 1.4787−02 | 8.4261−02 | 2.3694−01 | 1.1403 00 | 3.1013 00 | 3.5401 01 |
| 7.−05 | 0.0000 00 | 1.4786−02 | 8.4237−02 | 2.3686−01 | 1.1397 00 | 3.0997 00 | 3.0403 01 |
| 8.−05 | 0.0000 00 | 1.4786−02 | 8.4213−02 | 2.3679−01 | 1.1391 00 | 3.0981 00 | 2.6655 01 |
| 9.−05 | 0.0000 00 | 1.4785−02 | 8.4190−02 | 2.3672−01 | 1.1385 00 | 3.0965 00 | 2.3741 01 |
| 1.−04 | 0.0000 00 | 1.4785−02 | 8.4166−02 | 2.3664−01 | 1.1379 00 | 3.0948 00 | 2.1410 01 |
| 2.−04 | 0.0000 00 | 1.4781−02 | 8.3927−02 | 2.3591−01 | 1.1316 00 | 3.0748 00 | 1.0937 01 |
| 3.−04 | 0.0000 00 | 1.4777−02 | 8.3688−02 | 2.3517−01 | 1.1247 00 | 3.0472 00 | 7.4733 00 |
| 4.−04 | 0.0000 00 | 1.4773−02 | 8.3450−02 | 2.3443−01 | 1.1173 00 | 3.0076 00 | 5.7714 00 |
| 5.−04 | 0.0000 00 | 1.4769−02 | 8.3211−02 | 2.3369−01 | 1.1091 00 | 2.9485 00 | 4.7896 00 |
| 6.−04 | 0.0000 00 | 1.4765−02 | 8.2973−02 | 2.3294−01 | 1.1001 00 | 2.8598 00 | 4.1883 00 |
| 7.−04 | 0.0000 00 | 1.4761−02 | 8.2736−02 | 2.3220−01 | 1.0904 00 | 2.7344 00 | 3.8244 00 |
| 8.−04 | 0.0000 00 | 1.4757−02 | 8.2498−02 | 2.3145−01 | 1.0798 00 | 2.5810 00 | 3.6143 00 |
| 9.−04 | 0.0000 00 | 1.4753−02 | 8.2261−02 | 2.3070−01 | 1.0682 00 | 2.4213 00 | 3.4950 00 |
| 1.−03 | 0.0000 00 | 1.4749−02 | 8.2024−02 | 2.2995−01 | 1.0557 00 | 2.2721 00 | 3.4243 00 |

[a] Universal constants, $S_j$, in Eq. (7) calculated using $Pu^{239}$ delayed neutron data of reference 6. Parameter $\Lambda$ is prompt neutron lifetime. Floating decimal notation: 1.2345 03 ≡ 1234.5; 5.−08 ≡ $5 \times 10^{-8}$, etc. *All roots are negative: note that $-S_j$ values are tabulated.*

code we have taken $\delta n/n$ (relative change in $n$ per integration interval, $h$) as an indicator to dictate changes in $h$. The integration interval remains unchanged when $F_1 < |\delta n/n| < F_2$, (fiducials $F_1$ and $F_2$ specified in input) and is appropriately increased or decreased by a specified factor when $|\delta n/n|$ is outside this range. The testing sequence actually used in the code is given in Appendix A.

TABLE II-B

CHARACTERISTIC COEFFICIENTS, $A_j$, FOR $Pu^{239}$ [a]

| Λ | $A_0$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| 1.−08 | 3.2516 01 | 2.7487 00 | 2.8312 01 | 3.0828 01 | 4.8465 01 | 4.2291 01 | 9.9789 07 |
| 2.−08 | 3.2516 01 | 2.7487 00 | 2.8312 01 | 3.0828 01 | 4.8465 01 | 4.2292 01 | 4.9894 07 |
| 3.−08 | 3.2516 01 | 2.7487 00 | 2.8312 01 | 3.0828 01 | 4.8465 01 | 4.2293 01 | 3.3263 07 |
| 4.−08 | 3.2516 01 | 2.7487 00 | 2.8312 01 | 3.0828 01 | 4.8466 01 | 4.2294 01 | 2.4947 07 |
| 5.−08 | 3.2516 01 | 2.7487 00 | 2.8312 01 | 3.0828 01 | 4.8466 01 | 4.2295 01 | 1.9957 07 |
| 6.−08 | 3.2516 01 | 2.7487 00 | 2.8312 01 | 3.0828 01 | 4.8467 01 | 4.2296 01 | 1.6631 07 |
| 7.−08 | 3.2516 01 | 2.7487 00 | 2.8312 01 | 3.0828 01 | 4.8467 01 | 4.2298 01 | 1.4255 07 |
| 8.−08 | 3.2516 01 | 2.7487 00 | 2.8313 01 | 3.0828 01 | 4.8468 01 | 4.2299 01 | 1.2473 07 |
| 9.−08 | 3.2516 01 | 2.7487 00 | 2.8313 01 | 3.0828 01 | 4.8468 01 | 4.2300 01 | 1.1087 07 |
| 1.−07 | 3.2516 01 | 2.7487 00 | 2.8313 01 | 3.0828 01 | 4.8468 01 | 4.2301 01 | 9.9788 06 |
| 2.−07 | 3.2516 01 | 2.7487 00 | 2.8313 01 | 3.0828 01 | 4.8473 01 | 4.2312 01 | 4.9893 06 |
| 3.−07 | 3.2516 01 | 2.7487 00 | 2.8313 01 | 3.0829 01 | 4.8477 01 | 4.2322 01 | 3.3261 06 |
| 4.−07 | 3.2516 01 | 2.7487 00 | 2.8313 01 | 3.0829 01 | 4.8481 01 | 4.2333 01 | 2.4945 06 |
| 5.−07 | 3.2516 01 | 2.7487 00 | 2.8313 01 | 3.0829 01 | 4.8485 01 | 4.2344 01 | 1.9956 06 |
| 6.−07 | 3.2516 01 | 2.7487 00 | 2.8313 01 | 3.0829 01 | 4.8490 01 | 4.2355 01 | 1.6629 06 |
| 7.−07 | 3.2515 01 | 2.7487 00 | 2.8313 01 | 3.0830 01 | 4.8494 01 | 4.2366 01 | 1.4253 06 |
| 8.−07 | 3.2515 01 | 2.7487 00 | 2.8313 01 | 3.0830 01 | 4.8498 01 | 4.2377 01 | 1.2471 06 |
| 9.−07 | 3.2515 01 | 2.7487 00 | 2.8313 01 | 3.0830 01 | 4.8503 01 | 4.2388 01 | 1.1085 06 |
| 1.−06 | 3.2515 01 | 2.7486 00 | 2.8313 01 | 3.0830 01 | 4.8507 01 | 4.2399 01 | 9.9771 05 |
| 2.−06 | 3.2514 01 | 2.7485 00 | 2.8314 01 | 3.0832 01 | 4.8549 01 | 4.2508 01 | 4.9876 05 |
| 3.−06 | 3.2513 01 | 2.7485 00 | 2.8315 01 | 3.0834 01 | 4.8592 01 | 4.2617 01 | 3.3244 05 |
| 4.−06 | 3.2512 01 | 2.7484 00 | 2.8315 01 | 3.0836 01 | 4.8635 01 | 4.2727 01 | 2.4928 05 |
| 5.−06 | 3.2511 01 | 2.7483 00 | 2.8316 01 | 3.0839 01 | 4.8678 01 | 4.2838 01 | 1.9939 05 |
| 6.−06 | 3.2510 01 | 2.7482 00 | 2.8317 01 | 3.0841 01 | 4.8721 01 | 4.2949 01 | 1.6613 05 |
| 7.−06 | 3.2509 01 | 2.7481 00 | 2.8317 01 | 3.0843 01 | 4.8764 01 | 4.3060 01 | 1.4237 05 |
| 8.−06 | 3.2508 01 | 2.7480 00 | 2.8318 01 | 3.0845 01 | 4.8807 01 | 4.3172 01 | 1.2455 05 |
| 9.−06 | 3.2507 01 | 2.7479 00 | 2.8319 01 | 3.0847 01 | 4.8850 01 | 4.3284 01 | 1.1069 05 |
| 1.−05 | 3.2506 01 | 2.7478 00 | 2.8319 01 | 3.0849 01 | 4.8893 01 | 4.3396 01 | 9.9603 04 |
| 2.−05 | 3.2495 01 | 2.7468 00 | 2.8326 01 | 3.0870 01 | 4.9326 01 | 4.4544 01 | 4.9706 04 |
| 3.−05 | 3.2485 01 | 2.7458 00 | 2.8333 01 | 3.0891 01 | 4.9764 01 | 4.5737 01 | 3.3073 04 |
| 4.−05 | 3.2474 01 | 2.7448 00 | 2.8340 01 | 3.0912 01 | 5.0208 01 | 4.6976 01 | 2.4755 04 |
| 5.−05 | 3.2463 01 | 2.7438 00 | 2.8347 01 | 3.0933 01 | 5.0656 01 | 4.8264 01 | 1.9764 04 |
| 6.−05 | 3.2453 01 | 2.7428 00 | 2.8353 01 | 3.0954 01 | 5.1109 01 | 4.9603 01 | 1.6436 04 |
| 7.−05 | 3.2442 01 | 2.7418 00 | 2.8360 01 | 3.0974 01 | 5.1568 01 | 5.0996 01 | 1.4058 04 |
| 8.−05 | 3.2432 01 | 2.7409 00 | 2.8367 01 | 3.0995 01 | 5.2032 01 | 5.2446 01 | 1.2274 04 |
| 9.−05 | 3.2421 01 | 2.7399 00 | 2.8373 01 | 3.1016 01 | 5.2500 01 | 5.3955 01 | 1.0886 04 |
| 1.−04 | 3.2411 01 | 2.7389 00 | 2.8380 01 | 3.1036 01 | 5.2975 01 | 5.5526 01 | 9.7759 03 |
| 2.−04 | 3.2306 01 | 2.7291 00 | 2.8444 01 | 3.1237 01 | 5.8022 01 | 7.5522 01 | 4.7612 03 |
| 3.−04 | 3.2202 01 | 2.7193 00 | 2.8505 01 | 3.1431 01 | 6.3667 01 | 1.0707 02 | 3.0607 03 |
| 4.−04 | 3.2099 01 | 2.7096 00 | 2.8563 01 | 3.1618 01 | 6.9968 01 | 1.5878 02 | 2.1710 03 |
| 5.−04 | 3.1996 01 | 2.7000 00 | 2.8618 01 | 3.1797 01 | 7.6980 01 | 2.4410 02 | 1.5796 03 |
| 6.−04 | 3.1894 01 | 2.6903 00 | 2.8670 01 | 3.1967 01 | 8.4746 01 | 3.7295 02 | 1.1102 03 |
| 7.−04 | 3.1793 01 | 2.6808 00 | 2.8718 01 | 3.2130 01 | 9.3289 01 | 5.1992 02 | 7.1703 02 |
| 8.−04 | 3.1692 01 | 2.6712 00 | 2.8764 01 | 3.2284 01 | 1.0259 02 | 6.1951 02 | 4.2984 02 |
| 9.−04 | 3.1592 01 | 2.6617 00 | 2.8805 01 | 3.2429 01 | 1.1261 02 | 6.4434 02 | 2.5632 02 |
| 1.−03 | 3.1492 01 | 2.6523 00 | 2.8844 01 | 3.2565 01 | 1.2323 02 | 6.1896 02 | 1.6013 02 |

[a] Universal constants, $A_j$, in Eq. (7), calculated using $Pu^{239}$ delayed neutron data of reference *6*. Parameter Λ is prompt neutron lifetime. Floating decimal notation: 1.2345 03 ≡ 1234.5; 5.−08 ≡ 5 × $10^{-8}$, etc.

Representative solutions of Eq. (7) computed by the RTS code are plotted in Figs. 1 through 4. Figure 1 shows the $n(t)$ response to linear time variation of reactivity ($\delta k = at$) from initial equilibrium for assumed values of the prompt neutron generation time, $\Lambda = 10^{-5}$ and $10^{-4}$ sec.

Figure 2 shows $n(t)$ response for the elementary case of constant $k$; i.e., step change of reactivity

TABLE II-C

CHARACTERISTIC COEFFICIENTS, $B_j$, FOR $Pu^{239a}$

| $\Lambda$ | $B_0$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ |
|---|---|---|---|---|---|---|---|
| 1.−08 | 3.2516−07 | 2.7487−08 | 2.8312−07 | 3.0828−07 | 4.8465−07 | 4.2291−07 | 9.9999−01 |
| 2.−08 | 6.5033−07 | 5.4975−08 | 5.6625−07 | 6.1657−07 | 9.6930−07 | 8.4585−07 | 9.9999−01 |
| 3.−08 | 9.7550−07 | 8.2463−08 | 8.4938−07 | 9.2485−07 | 1.4539−06 | 1.2688−06 | 9.9999−01 |
| 4.−08 | 1.3006−06 | 1.0995−07 | 1.1325−06 | 1.2331−06 | 1.9386−06 | 1.6917−06 | 9.9999−01 |
| 5.−08 | 1.6258−06 | 1.3743−07 | 1.4156−06 | 1.5414−06 | 2.4233−06 | 2.1147−06 | 9.9999−01 |
| 6.−08 | 1.9509−06 | 1.6492−07 | 1.6987−06 | 1.8497−06 | 2.9080−06 | 2.5378−06 | 9.9998−01 |
| 7.−08 | 2.2761−06 | 1.9241−07 | 1.9819−06 | 2.1580−06 | 3.3927−06 | 2.9608−06 | 9.9998−01 |
| 8.−08 | 2.6013−06 | 2.1990−07 | 2.2650−06 | 2.4662−06 | 3.8774−06 | 3.3839−06 | 9.9998−01 |
| 9.−08 | 2.9264−06 | 2.4739−07 | 2.5481−06 | 2.7745−06 | 4.3621−06 | 3.8070−06 | 9.9998−01 |
| 1.−07 | 3.2516−06 | 2.7487−07 | 2.8313−06 | 3.0828−06 | 4.8468−06 | 4.2301−06 | 9.9998−01 |
| 2.−07 | 6.5033−06 | 5.4975−07 | 5.6626−06 | 6.1657−06 | 9.6946−06 | 8.4624−06 | 9.9996−01 |
| 3.−07 | 9.7549−06 | 8.2463−07 | 8.4939−06 | 9.2487−06 | 1.4543−05 | 1.2696−05 | 9.9994−01 |
| 4.−07 | 1.3006−05 | 1.0995−06 | 1.1325−05 | 1.2331−05 | 1.9392−05 | 1.6933−05 | 9.9992−01 |
| 5.−07 | 1.6258−05 | 1.3743−06 | 1.4156−05 | 1.5414−05 | 2.4243−05 | 2.1172−05 | 9.9990−01 |
| 6.−07 | 1.9509−05 | 1.6492−06 | 1.6988−05 | 1.8497−05 | 2.9094−05 | 2.5413−05 | 9.9988−01 |
| 7.−07 | 2.2761−05 | 1.9241−06 | 1.9819−05 | 2.1581−05 | 3.3946−05 | 2.9656−05 | 9.9987−01 |
| 8.−07 | 2.6012−05 | 2.1989−06 | 2.2650−05 | 2.4664−05 | 3.8799−05 | 3.3901−05 | 9.9985−01 |
| 9.−07 | 2.9264−05 | 2.4738−06 | 2.5482−05 | 2.7747−05 | 4.3652−05 | 3.8149−05 | 9.9983−01 |
| 1.−06 | 3.2515−05 | 2.7486−06 | 2.8313−05 | 3.0830−05 | 4.8507−05 | 4.2399−05 | 9.9981−01 |
| 2.−06 | 6.5029−05 | 5.4971−06 | 5.6628−05 | 6.1665−05 | 9.7100−05 | 8.5017−05 | 9.9962−01 |
| 3.−06 | 9.7540−05 | 8.2455−06 | 8.4945−05 | 9.2504−05 | 1.4577−04 | 1.2785−04 | 9.9944−01 |
| 4.−06 | 1.3004−04 | 1.0993−05 | 1.1326−04 | 1.2334−04 | 1.9454−04 | 1.7091−04 | 9.9925−01 |
| 5.−06 | 1.6255−04 | 1.3741−05 | 1.4158−04 | 1.5419−04 | 2.4339−04 | 2.1419−04 | 9.9907−01 |
| 6.−06 | 1.9506−04 | 1.6489−05 | 1.6990−04 | 1.8504−04 | 2.9232−04 | 2.5769−04 | 9.9888−01 |
| 7.−06 | 2.2756−04 | 1.9236−05 | 1.9822−04 | 2.1590−04 | 3.4135−04 | 3.0142−04 | 9.9869−01 |
| 8.−06 | 2.6006−04 | 2.1984−05 | 2.2654−04 | 2.4676−04 | 3.9046−04 | 3.4538−04 | 9.9850−01 |
| 9.−06 | 2.9256−04 | 2.4731−05 | 2.5487−04 | 2.7762−04 | 4.3965−04 | 3.8956−04 | 9.9832−01 |
| 1.−05 | 3.2596−04 | 2.7478−05 | 2.8319−04 | 3.0849−04 | 4.8893−04 | 4.3397−04 | 9.9813−01 |
| 2.−05 | 6.4991−04 | 5.4936−05 | 5.6653−04 | 6.1741−04 | 9.8655−04 | 8.9095−04 | 9.9623−01 |
| 3.−05 | 9.7455−04 | 8.2375−05 | 8.5000−04 | 9.2675−04 | 1.4929−03 | 1.3722−03 | 9.9430−01 |
| 4.−05 | 1.2989−03 | 1.0979−04 | 1.1336−03 | 1.2365−03 | 2.0084−03 | 1.8793−03 | 9.9233−01 |
| 5.−05 | 1.6231−03 | 1.3719−04 | 1.4173−03 | 1.5466−03 | 2.5329−03 | 2.4136−03 | 9.9032−01 |
| 6.−05 | 1.9472−03 | 1.6457−04 | 1.7012−03 | 1.8572−03 | 3.0667−03 | 2.9767−03 | 9.8828−01 |
| 7.−05 | 2.2710−03 | 1.9193−04 | 1.9852−03 | 2.1682−03 | 3.6100−03 | 3.5705−03 | 9.8620−01 |
| 8.−05 | 2.5945−03 | 2.1927−04 | 2.2693−03 | 2.4796−03 | 4.1629−03 | 4.1967−03 | 9.8407−01 |
| 9.−05 | 2.9179−03 | 2.4659−04 | 2.5536−03 | 2.7915−03 | 4.7255−03 | 4.8573−03 | 9.8190−01 |
| 1.−04 | 3.2411−03 | 2.7389−04 | 2.8380−03 | 3.1037−03 | 5.2981−03 | 5.5544−03 | 9.7969−01 |
| 2.−04 | 6.4613−03 | 5.4583−04 | 5.6890−03 | 6.2478−03 | 1.1607−02 | 1.5113−02 | 9.5433−01 |
| 3.−04 | 9.6607−03 | 8.1581−04 | 8.5520−03 | 9.4302−03 | 1.9106−02 | 3.2150−02 | 9.2028−01 |
| 4.−04 | 1.2839−02 | 1.0838−03 | 1.1425−02 | 1.2648−02 | 2.7999−02 | 6.3589−02 | 8.7041−01 |
| 5.−04 | 1.5998−02 | 1.3500−03 | 1.4309−02 | 1.5900−02 | 3.8511−02 | 1.2223−01 | 7.9169−01 |
| 6.−04 | 1.9136−02 | 1.6142−03 | 1.7203−02 | 1.9183−02 | 5.0881−02 | 2.2415−01 | 6.6782−01 |
| 7.−04 | 2.2255−02 | 1.8765−03 | 2.0104−02 | 2.2494−02 | 6.5352−02 | 3.6464−01 | 5.0327−01 |
| 8.−04 | 2.5353−02 | 2.1370−03 | 2.3012−02 | 2.5832−02 | 8.2149−02 | 4.9663−01 | 3.4487−01 |
| 9.−04 | 2.8432−02 | 2.3956−03 | 2.5927−02 | 2.9192−02 | 1.0145−01 | 5.8117−01 | 2.3142−01 |
| 1.−03 | 3.1492−02 | 2.6523−03 | 2.8847−02 | 3.2573−02 | 1.2336−01 | 6.2037−01 | 1.6068−01 |

[a] Universal constants $B_j$, in Eq. (7), calculated using $Pu^{239}$ delayed neutron data of reference *6*. Parameter $\Lambda$ is prompt neutron lifetime. Floating decimal notation: 1.2345 03 ≡ 1234.5; 5.−08 ≡ $5 \times 10^{-8}$, etc.

$\delta k_0$ from initial equilibrium in $U^{235}$, $Pu^{239}$, and $U^{233}$ systems. For $\delta k_0 \leqq \beta$, response characteristics are clearly dominated by delayed neutron characteristics whereas for $\delta k_0 \gg \beta$, transient response becomes essentially independent of fissile species. Another family of solutions of Eq. (7) is given in Fig. 3 for quadratic time variation of reactivity in $U^{235}$ systems. Here we take $\Lambda = 10^{-4}$ sec and $\delta k(t) = at +$

TABLE III-A

CHARACTERISTIC ROOTS, $S_i$, FOR $U^{233}$ [a]

| Λ | $-S_0$ | $-S_1$ | $-S_2$ | $-S_3$ | $-S_4$ | $-S_5$ | $-S_6$ |
|---|---|---|---|---|---|---|---|
| 1.−08 | 0.0000 00 | 1.6940−02 | 8.5142−02 | 2.2562−01 | 1.1863 00 | 3.0645 00 | 2.7000 05 |
| 2.−08 | 0.0000 00 | 1.6940−02 | 8.5142−02 | 2.2562−01 | 1.1863 00 | 3.0645 00 | 1.3500 05 |
| 3.−08 | 0.0000 00 | 1.6940−02 | 8.5142−02 | 2.2562−01 | 1.1863 00 | 3.0645 00 | 9.0000 04 |
| 4.−08 | 0.0000 00 | 1.6940−02 | 8.5141−02 | 2.2562−01 | 1.1863 00 | 3.0645 00 | 6.7500 04 |
| 5.−08 | 0.0000 00 | 1.6940−02 | 8.5141−02 | 2.2562−01 | 1.1863 00 | 3.0645 00 | 5.4000 04 |
| 6.−08 | 0.0000 00 | 1.6940−02 | 8.5141−02 | 2.2562−01 | 1.1863 00 | 3.0645 00 | 4.5000 04 |
| 7.−08 | 0.0000 00 | 1.6940−02 | 8.5141−02 | 2.2562−01 | 1.1863 00 | 3.0645 00 | 3.8571 04 |
| 8.−08 | 0.0000 00 | 1.6940−02 | 8.5141−02 | 2.2562−01 | 1.1863 00 | 3.0645 00 | 3.3750 04 |
| 9.−08 | 0.0000 00 | 1.6940−02 | 8.5141−02 | 2.2562−01 | 1.1863 00 | 3.0645 00 | 3.0000 04 |
| 1.−07 | 0.0000 00 | 1.6940−02 | 8.5141−02 | 2.2562−01 | 1.1863 00 | 3.0645 00 | 2.7000 04 |
| 2.−07 | 0.0000 00 | 1.6940−02 | 8.5141−02 | 2.2562−01 | 1.1863 00 | 3.0645 00 | 1.3500 04 |
| 3.−07 | 0.0000 00 | 1.6940−02 | 8.5141−02 | 2.2562−01 | 1.1863 00 | 3.0645 00 | 9.0003 03 |
| 4.−07 | 0.0000 00 | 1.6940−02 | 8.5141−02 | 2.2561−01 | 1.1863 00 | 3.0645 00 | 6.7503 03 |
| 5.−07 | 0.0000 00 | 1.6940−02 | 8.5141−02 | 2.2561−01 | 1.1863 00 | 3.0645 00 | 5.4003 03 |
| 6.−07 | 0.0000 00 | 1.6940−02 | 8.5141−02 | 2.2561−01 | 1.1863 00 | 3.0645 00 | 4.5003 03 |
| 7.−07 | 0.0000 00 | 1.6940−02 | 8.5140−02 | 2.2561−01 | 1.1863 00 | 3.0644 00 | 3.8574 03 |
| 8.−07 | 0.0000 00 | 1.6940−02 | 8.5140−02 | 2.2561−01 | 1.1863 00 | 3.0644 00 | 3.3753 03 |
| 9.−07 | 0.0000 00 | 1.6940−02 | 8.5140−02 | 2.2561−01 | 1.1863 00 | 3.0644 00 | 3.0003 03 |
| 1.−06 | 0.0000 00 | 1.6940−02 | 8.5140−02 | 2.2561−01 | 1.1863 00 | 3.0644 00 | 2.7003 03 |
| 2.−06 | 0.0000 00 | 1.6940−02 | 8.5138−02 | 2.2561−01 | 1.1862 00 | 3.0644 00 | 1.3503 03 |
| 3.−06 | 0.0000 00 | 1.6940−02 | 8.5137−02 | 2.2560−01 | 1.1862 00 | 3.0643 00 | 9.0030 02 |
| 4.−06 | 0.0000 00 | 1.6939−02 | 8.5135−02 | 2.2560−01 | 1.1862 00 | 3.0642 00 | 6.7530 02 |
| 5.−06 | 0.0000 00 | 1.6939−02 | 8.5134−02 | 2.2560−01 | 1.1861 00 | 3.0642 00 | 5.4030 02 |
| 6.−06 | 0.0000 00 | 1.6939−02 | 8.5132−02 | 2.2559−01 | 1.1861 00 | 3.0641 00 | 4.5030 02 |
| 7.−06 | 0.0000 00 | 1.6939−02 | 8.5130−02 | 2.2559−01 | 1.1861 00 | 3.0640 00 | 3.8601 02 |
| 8.−06 | 0.0000 00 | 1.6939−02 | 8.5129−02 | 2.2558−01 | 1.1860 00 | 3.0640 00 | 3.3780 02 |
| 9.−06 | 0.0000 00 | 1.6939−02 | 8.5127−02 | 2.2558−01 | 1.1860 00 | 3.0639 00 | 3.0030 02 |
| 1.−05 | 0.0000 00 | 1.6939−02 | 8.5126−02 | 2.2557−01 | 1.1860 00 | 3.0638 00 | 2.7030 02 |
| 2.−05 | 0.0000 00 | 1.6938−02 | 8.5110−02 | 2.2553−01 | 1.1856 00 | 3.0631 00 | 1.3530 02 |
| 3.−05 | 0.0000 00 | 1.6938−02 | 8.5094−02 | 2.2549−01 | 1.1853 00 | 3.0624 00 | 9.0303 01 |
| 4.−05 | 0.0000 00 | 1.6937−02 | 8.5078−02 | 2.2545−01 | 1.1850 00 | 3.0617 00 | 6.7804 01 |
| 5.−05 | 0.0000 00 | 1.6936−02 | 8.5062−02 | 2.2541−01 | 1.1846 00 | 3.0609 00 | 5.4305 01 |
| 6.−05 | 0.0000 00 | 1.6936−02 | 8.5046−02 | 2.2537−01 | 1.1843 00 | 3.0602 00 | 4.5307 01 |
| 7.−05 | 0.0000 00 | 1.6935−02 | 8.5030−02 | 2.2532−01 | 1.1839 00 | 3.0594 00 | 3.8879 01 |
| 8.−05 | 0.0000 00 | 1.6934−02 | 8.5015−02 | 2.2528−01 | 1.1836 00 | 3.0586 00 | 3.4059 01 |
| 9.−05 | 0.0000 00 | 1.6934−02 | 8.4999−02 | 2.2524−01 | 1.1833 00 | 3.0578 00 | 3.0310 01 |
| 1.−04 | 0.0000 00 | 1.6933−02 | 8.4983−02 | 2.2520−01 | 1.1829 00 | 3.0570 00 | 2.7311 01 |
| 2.−04 | 0.0000 00 | 1.6927−02 | 8.4824−02 | 2.2478−01 | 1.1792 00 | 3.0477 00 | 1.3825 01 |
| 3.−04 | 0.0000 00 | 1.6920−02 | 8.4665−02 | 2.2436−01 | 1.1753 00 | 3.0357 00 | 9.3419 00 |
| 4.−04 | 0.0000 00 | 1.6913−02 | 8.4507−02 | 2.2394−01 | 1.1711 00 | 3.0200 00 | 7.1125 00 |
| 5.−04 | 0.0000 00 | 1.6907−02 | 8.4348−02 | 2.2352−01 | 1.1665 00 | 2.9985 00 | 5.7891 00 |
| 6.−04 | 0.0000 00 | 1.6900−02 | 8.4189−02 | 2.2310−01 | 1.1615 00 | 2.9681 00 | 4.9251 00 |
| 7.−04 | 0.0000 00 | 1.6894−02 | 8.4030−02 | 2.2267−01 | 1.1561 00 | 2.9236 00 | 4.3328 00 |
| 8.−04 | 0.0000 00 | 1.6887−02 | 8.3872−02 | 2.2224−01 | 1.1502 00 | 2.8571 00 | 3.9236 00 |
| 9.−04 | 0.0000 00 | 1.6881−02 | 8.3713−02 | 2.2182−01 | 1.1438 00 | 2.7617 00 | 3.6509 00 |
| 1.−03 | 0.0000 00 | 1.6874−02 | 8.3554−02 | 2.2139−01 | 1.1368 00 | 2.6395 00 | 3.4807 00 |

[a] Universal constants, $S_i$, in Eq. (7), calculated using $U^{233}$ delayed neutron data of reference 6. Parameter Λ is prompt neutron lifetime. Floating decimal notation: 1.2345 03 ≡ 1234.5; 5.−08 ≡ 5 × $10^{-8}$, etc. *All roots are negative: note that* $-S_i$ *values are tabulated.*

$bt^2$ with $a$ and $b$ variable. Setting $a$ equal to zero corresponds to free-fall displacement of a linear control rod; damped control rod motion can be approximated by suitable choice of (negative) $a$.

In connection with problems of reactor startup and shutdown, the $n(t)$ or power response to various programmed $\delta k(t)$ functions have been investigated with the RTS code. As one example, Fig. 4 shows

TABLE III-B

CHARACTERISTIC COEFFICIENTS, $A_j$, FOR $U^{233}$ [a]

| Λ | $A_0$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ |
|---|---|---|---|---|---|---|---|
| 1.−08 | 2.0718 01 | 3.9055 00 | 1.8635 01 | 1.8429 01 | 2.7590 01 | 2.1982 01 | 9.9729 07 |
| 2.−08 | 2.0718 01 | 3.9055 00 | 1.8635 01 | 1.8429 01 | 2.7590 01 | 2.1983 01 | 4.9864 07 |
| 3.−08 | 2.0718 01 | 3.9055 00 | 1.8635 01 | 1.8429 01 | 2.7591 01 | 2.1983 01 | 3.3243 07 |
| 4.−08 | 2.0718 01 | 3.9055 00 | 1.8635 01 | 1.8429 01 | 2.7591 01 | 2.1984 01 | 2.4932 07 |
| 5.−08 | 2.0718 01 | 3.9055 00 | 1.8635 01 | 1.8429 01 | 2.7591 01 | 2.1984 01 | 1.9945 07 |
| 6.−08 | 2.0718 01 | 3.9055 00 | 1.8635 01 | 1.8429 01 | 2.7591 01 | 2.1984 01 | 1.6621 07 |
| 7.−08 | 2.0718 01 | 3.9055 00 | 1.8635 01 | 1.8429 01 | 2.7591 01 | 2.1985 01 | 1.4247 07 |
| 8.−08 | 2.0718 01 | 3.9055 00 | 1.8635 01 | 1.8429 01 | 2.7592 01 | 2.1985 01 | 1.2466 07 |
| 9.−08 | 2.0718 01 | 3.9055 00 | 1.8635 01 | 1.8429 01 | 2.7592 01 | 2.1986 01 | 1.1080 07 |
| 1.−07 | 2.0718 01 | 3.9055 00 | 1.8635 01 | 1.8429 01 | 2.7592 01 | 2.1986 01 | 9.9728 06 |
| 2.−07 | 2.0718 01 | 3.9055 00 | 1.8635 01 | 1.8429 01 | 2.7594 01 | 2.1991 01 | 4.9863 06 |
| 3.−07 | 2.0718 01 | 3.9054 00 | 1.8635 01 | 1.8429 01 | 2.7596 01 | 2.1995 01 | 3.3242 06 |
| 4.−07 | 2.0718 01 | 3.9054 00 | 1.8635 01 | 1.8429 01 | 2.7598 01 | 2.2000 01 | 2.4931 06 |
| 5.−07 | 2.0718 01 | 3.9054 00 | 1.8635 01 | 1.8429 01 | 2.7600 01 | 2.2004 01 | 1.9944 06 |
| 6.−07 | 2.0718 01 | 3.9054 00 | 1.8635 01 | 1.8429 01 | 2.7602 01 | 2.2009 01 | 1.6620 06 |
| 7.−07 | 2.0718 01 | 3.9054 00 | 1.8635 01 | 1.8429 01 | 2.7604 01 | 2.2013 01 | 1.4246 06 |
| 8.−07 | 2.0718 01 | 3.9054 00 | 1.8635 01 | 1.8429 01 | 2.7606 01 | 2.2018 01 | 1.2465 06 |
| 9.−07 | 2.0718 01 | 3.9054 00 | 1.8635 01 | 1.8430 01 | 2.7608 01 | 2.2022 01 | 1.1079 06 |
| 1.−06 | 2.0718 01 | 3.9054 00 | 1.8636 01 | 1.8430 01 | 2.7610 01 | 2.2026 01 | 9.9718 05 |
| 2.−06 | 2.0717 01 | 3.9053 00 | 1.8636 01 | 1.8431 01 | 2.7631 01 | 2.2071 01 | 4.9853 05 |
| 3.−06 | 2.0717 01 | 3.9052 00 | 1.8636 01 | 1.8432 01 | 2.7651 01 | 2.2116 01 | 3.3232 05 |
| 4.−06 | 2.0716 01 | 3.9052 00 | 1.8637 01 | 1.8433 01 | 2.7672 01 | 2.2161 01 | 2.4921 05 |
| 5.−06 | 2.0716 01 | 3.9051 00 | 1.8637 01 | 1.8434 01 | 2.7693 01 | 2.2206 01 | 1.9934 05 |
| 6.−06 | 2.0715 01 | 3.9050 00 | 1.8637 01 | 1.8435 01 | 2.7713 01 | 2.2252 01 | 1.6610 05 |
| 7.−06 | 2.0715 01 | 3.9049 00 | 1.8638 01 | 1.8436 01 | 2.7734 01 | 2.2297 01 | 1.4235 05 |
| 8.−06 | 2.0715 01 | 3.9049 00 | 1.8638 01 | 1.8437 01 | 2.7755 01 | 2.2343 01 | 1.2455 05 |
| 9.−06 | 2.0714 01 | 3.9048 00 | 1.8639 01 | 1.8438 01 | 2.7775 01 | 2.2388 01 | 1.1069 05 |
| 1.−05 | 2.0714 01 | 3.9047 00 | 1.8639 01 | 1.8439 01 | 2.7796 01 | 2.2434 01 | 9.9618 04 |
| 2.−05 | 2.0709 01 | 3.9039 00 | 1.8643 01 | 1.8449 01 | 2.8004 01 | 2.2900 01 | 4.9752 04 |
| 3.−05 | 2.0705 01 | 3.9032 00 | 1.8647 01 | 1.8459 01 | 2.8214 01 | 2.3381 01 | 3.3130 04 |
| 4.−05 | 2.0701 01 | 3.9024 00 | 1.8651 01 | 1.8469 01 | 2.8426 01 | 2.3876 01 | 2.4818 04 |
| 5.−05 | 2.0697 01 | 3.9017 00 | 1.8655 01 | 1.8479 01 | 2.8641 01 | 2.4387 01 | 1.9831 04 |
| 6.−05 | 2.0692 01 | 3.9009 00 | 1.8658 01 | 1.8489 01 | 2.8858 01 | 2.4914 01 | 1.6506 04 |
| 7.−05 | 2.0688 01 | 3.9001 00 | 1.8662 01 | 1.8499 01 | 2.9077 01 | 2.5458 01 | 1.4130 04 |
| 8.−05 | 2.0684 01 | 3.8994 00 | 1.8666 01 | 1.8510 01 | 2.9298 01 | 2.6019 01 | 1.2349 04 |
| 9.−05 | 2.0679 01 | 3.8986 00 | 1.8670 01 | 1.8520 01 | 2.9521 01 | 2.6598 01 | 1.0963 04 |
| 1.−04 | 2.0675 01 | 3.8979 00 | 1.8674 01 | 1.8530 01 | 2.9747 01 | 2.7197 01 | 9.8542 03 |
| 2.−04 | 2.0633 01 | 3.8902 00 | 1.8712 01 | 1.8629 01 | 3.2138 01 | 3.4424 01 | 4.8580 03 |
| 3.−04 | 2.0590 01 | 3.8826 00 | 1.8748 01 | 1.8727 01 | 3.4792 01 | 4.4779 01 | 3.1828 03 |
| 4.−04 | 2.0548 01 | 3.8751 00 | 1.8784 01 | 1.8824 01 | 3.7741 01 | 6.0182 01 | 2.3332 03 |
| 5.−04 | 2.0506 01 | 3.8675 00 | 1.8819 01 | 1.8918 01 | 4.1023 01 | 8.4037 01 | 1.8074 03 |
| 6.−04 | 2.0464 01 | 3.8599 00 | 1.8853 01 | 1.9011 01 | 4.4676 01 | 1.2234 02 | 1.4329 03 |
| 7.−04 | 2.0422 01 | 3.8524 00 | 1.8885 01 | 1.9103 01 | 4.8743 01 | 1.8461 02 | 1.1290 03 |
| 8.−04 | 2.0380 01 | 3.8448 00 | 1.8917 01 | 1.9192 01 | 5.3267 01 | 2.7993 02 | 8.5109 02 |
| 9.−04 | 2.0339 01 | 3.8373 00 | 1.8948 01 | 1.9280 01 | 5.8291 01 | 3.9833 02 | 5.8908 02 |
| 1.−03 | 2.0297 01 | 3.8297 00 | 1.8978 01 | 1.9365 01 | 6.3856 01 | 4.9724 02 | 3.7372 02 |

[a] Universal constants, $A_j$, in Eq. (7), calculated using $U^{233}$ delayed neutron data of reference 6. Parameter Λ is prompt neutron lifetime. Floating decimal notation: 1.2345 03 ≡ 1234.5; 5.−08 ≡ 5 × $10^{-8}$, etc.

parametric power response curves for the indicated $\delta k(t)$ function, designed to give steady power increase to a predetermined "power plateau," after which reactor power remains essentially constant.

Nearly any analytic function for $\delta k(t)$ variation can be inserted into the RTS code; in cases where the $\delta k(t)$ variation is not readily expressed analytically, it may be specified numerically as a point-

TABLE III-C

CHARACTERISTIC COEFFICIENTS, $B_j$, FOR $U^{233}$ [a]

| Λ | $B_0$ | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ |
|---|---|---|---|---|---|---|---|
| 1. −08 | 2.0718−07 | 3.9055−08 | 1.8635−07 | 1.8429−07 | 2.7590−07 | 2.1982−07 | 9.9999 01 |
| 2. −08 | 4.1436−07 | 7.8110−08 | 3.7271−07 | 3.6858−07 | 5.5181−07 | 4.3966−07 | 9.9999−01 |
| 3. −08 | 6.2155−07 | 1.1716−07 | 5.5906−07 | 5.5287−07 | 8.2773−07 | 6.5950−07 | 9.9999−01 |
| 4. −08 | 8.2873−07 | 1.5622−07 | 7.4542−07 | 7.3716−07 | 1.1036−06 | 8.7936−07 | 9.9999−01 |
| 5. −08 | 1.0359−06 | 1.9527−07 | 9.3178−07 | 9.2145−07 | 1.3795−06 | 1.0992−06 | 9.9999−01 |
| 6. −08 | 1.2431−06 | 2.3433−07 | 1.1181−06 | 1.1057−06 | 1.6555−06 | 1.3190−06 | 9.9999−01 |
| 7. −08 | 1.4502−06 | 2.7338−07 | 1.3044−06 | 1.2900−06 | 1.9314−06 | 1.5389−06 | 9.9999−01 |
| 8. −08 | 1.6574−06 | 3.1244−07 | 1.4908−06 | 1.4743−06 | 2.2073−06 | 1.7588−06 | 9.9999−01 |
| 9. −08 | 1.8646−06 | 3.5149−07 | 1.6772−06 | 1.6586−06 | 2.4833−06 | 1.9787−06 | 9.9998−01 |
| 1. −07 | 2.0718−06 | 3.9055−07 | 1.8635−06 | 1.8429−06 | 2.7592−06 | 2.1986−06 | 9.9998−01 |
| 2. −07 | 4.1436−06 | 7.8110−07 | 3.7271−06 | 3.6858−06 | 5.5189−06 | 4.3982−06 | 9.9997−01 |
| 3. −07 | 6.2155−06 | 1.1716−06 | 5.5907−06 | 5.5288−06 | 8.2789−06 | 6.5987−06 | 9.9996−01 |
| 4. −07 | 8.2873−06 | 1.5621−06 | 7.4543−06 | 7.3718−06 | 1.1039−05 | 8.8000−06 | 9.9995−01 |
| 5. −07 | 1.0359−05 | 1.9527−06 | 9.3179−06 | 9.2148−06 | 1.3800−05 | 1.1002−05 | 9.9994−01 |
| 6. −07 | 1.2430−05 | 2.3432−06 | 1.1181−05 | 1.1057−05 | 1.6561−05 | 1.3205−05 | 9.9993−01 |
| 7. −07 | 1.4502−05 | 2.7338−06 | 1.3045−05 | 1.2900−05 | 1.9323−05 | 1.5409−05 | 9.9992−01 |
| 8. −07 | 1.6574−05 | 3.1243−06 | 1.4908−05 | 1.4743−05 | 2.2085−05 | 1.7614−05 | 9.9991−01 |
| 9. −07 | 1.8646−05 | 3.5149−06 | 1.6772−05 | 1.6587−05 | 2.4848−05 | 1.9820−05 | 9.9989−01 |
| 1. −06 | 2.0718−05 | 3.9054−06 | 1.8636−05 | 1.8430−05 | 2.7610−05 | 2.2027−05 | 9.9988−01 |
| 2. −06 | 4.1435−05 | 7.8107−06 | 3.7272−05 | 3.6862−05 | 5.5263−05 | 4.4143−05 | 9.9977−01 |
| 3. −06 | 6.2151−05 | 1.1715−05 | 5.5910−05 | 5.5296−05 | 8.2956−05 | 6.6350−05 | 9.9966−01 |
| 4. −06 | 8.2867−05 | 1.5620−05 | 7.4548−05 | 7.3732−05 | 1.1069−04 | 8.8647−05 | 9.9955−01 |
| 5. −06 | 1.0358−04 | 1.9525−05 | 9.3187−05 | 9.2171−05 | 1.3846−04 | 1.1103−04 | 9.9944−01 |
| 6. −06 | 1.2429−04 | 2.3430−05 | 1.1182−04 | 1.1061−04 | 1.6628−04 | 1.3351−04 | 9.9933−01 |
| 7. −06 | 1.4500−04 | 2.7334−05 | 1.3046−04 | 1.2905−04 | 1.9414−04 | 1.5608−04 | 9.9921−01 |
| 8. −06 | 1.6572−04 | 3.1239−05 | 1.4911−04 | 1.4749−04 | 2.2204−04 | 1.7874−04 | 9.9910−01 |
| 9. −06 | 1.8643−04 | 3.5143−05 | 1.6775−04 | 1.6594−04 | 2.4998−04 | 2.0150−04 | 9.9899−01 |
| 1. −05 | 2.0714−04 | 3.9047−05 | 1.8639−04 | 1.8439−04 | 2.7796−04 | 2.2435−04 | 9.9888−01 |
| 2. −05 | 4.1419−04 | 7.8079−05 | 3.7286−04 | 3.6899−04 | 5.6010−04 | 4.5804−04 | 9.9774−01 |
| 3. −05 | 6.2116−04 | 1.1709−04 | 5.5942−04 | 5.5379−04 | 8.4646−04 | 7.0150−04 | 9.9660−01 |
| 4. −05 | 8.2805−04 | 1.5609−04 | 7.4605−04 | 7.3879−04 | 1.1371−03 | 9.5518−04 | 9.9543−01 |
| 5. −05 | 1.0348−03 | 1.9508−04 | 9.3275−04 | 9.2400−04 | 1.4321−03 | 1.2195−03 | 9.9426−01 |
| 6. −05 | 1.2415−03 | 2.3405−04 | 1.1195−03 | 1.1094−03 | 1.7316−03 | 1.4951−03 | 9.9306−01 |
| 7. −05 | 1.4481−03 | 2.7301−04 | 1.3064−03 | 1.2950−03 | 2.0355−03 | 1.7824−03 | 9.9185−01 |
| 8. −05 | 1.6547−03 | 3.1195−04 | 1.4933−03 | 1.4808−03 | 2.3440−03 | 2.0820−03 | 9.9063−01 |
| 9. −05 | 1.8611−03 | 3.5088−04 | 1.6803−03 | 1.6668−03 | 2.6572−03 | 2.3945−03 | 9.8938−01 |
| 1. −04 | 2.0675−03 | 3.8979−04 | 1.8674−03 | 1.8530−03 | 2.9751−03 | 2.7205−03 | 9.8812−01 |
| 2. −04 | 4.1266−03 | 7.7806−04 | 3.7424−03 | 3.7260−03 | 6.4291−03 | 6.8891−03 | 9.7430−01 |
| 3. −04 | 6.1771−03 | 1.1648−03 | 5.6247−03 | 5.6186−03 | 1.0441−02 | 1.3446−02 | 9.5752−01 |
| 4. −04 | 8.2192−03 | 1.5500−03 | 7.5140−03 | 7.5302−03 | 1.5103−02 | 2.4102−02 | 9.3598−01 |
| 5. −04 | 1.0253−02 | 1.9337−03 | 9.4100−03 | 9.4606−03 | 2.0523−02 | 4.2081−02 | 9.0633−01 |
| 6. −04 | 1.2278−02 | 2.3159−03 | 1.1312−02 | 1.1408−02 | 2.6824−02 | 7.3538−02 | 8.6232−01 |
| 7. −04 | 1.4295−02 | 2.6967−03 | 1.3220−02 | 1.3374−02 | 3.4148−02 | 1.2949−01 | 7.9277−01 |
| 8. −04 | 1.6304−02 | 3.0759−03 | 1.5135−02 | 1.5356−02 | 4.2653−02 | 2.2445−01 | 6.8301−01 |
| 9. −04 | 1.8305−02 | 3.4536−03 | 1.7054−02 | 1.7355−02 | 5.2516−02 | 3.5939−01 | 5.3192−10 |
| 1. −03 | 2.0297−02 | 3.8298−03 | 1.8979−02 | 1.9370−02 | 6.3929−02 | 4.9855−01 | 3.7503−01 |

[a] Universal constants, $B_j$, in Eq. 7, calculated using $U^{233}$ delayed neutron data of reference *6*. Parameter Λ is prompt neutron lifetime. Floating decimal notation: 1.2345 03 ≡ 1234.5; 5. − 08 ≡ 5 × $10^{-8}$, etc.

function of time and fed into the code in digital form. The code as presently written can accommodate in a single run up to 500 data points, which need not be equally spaced in time. This allows one to follow transient response during arbitrary changes and/or programmed discontinuities in $\delta k(t)$. If more than 500 points should be required the problem can be continued by employing a "stop-start"

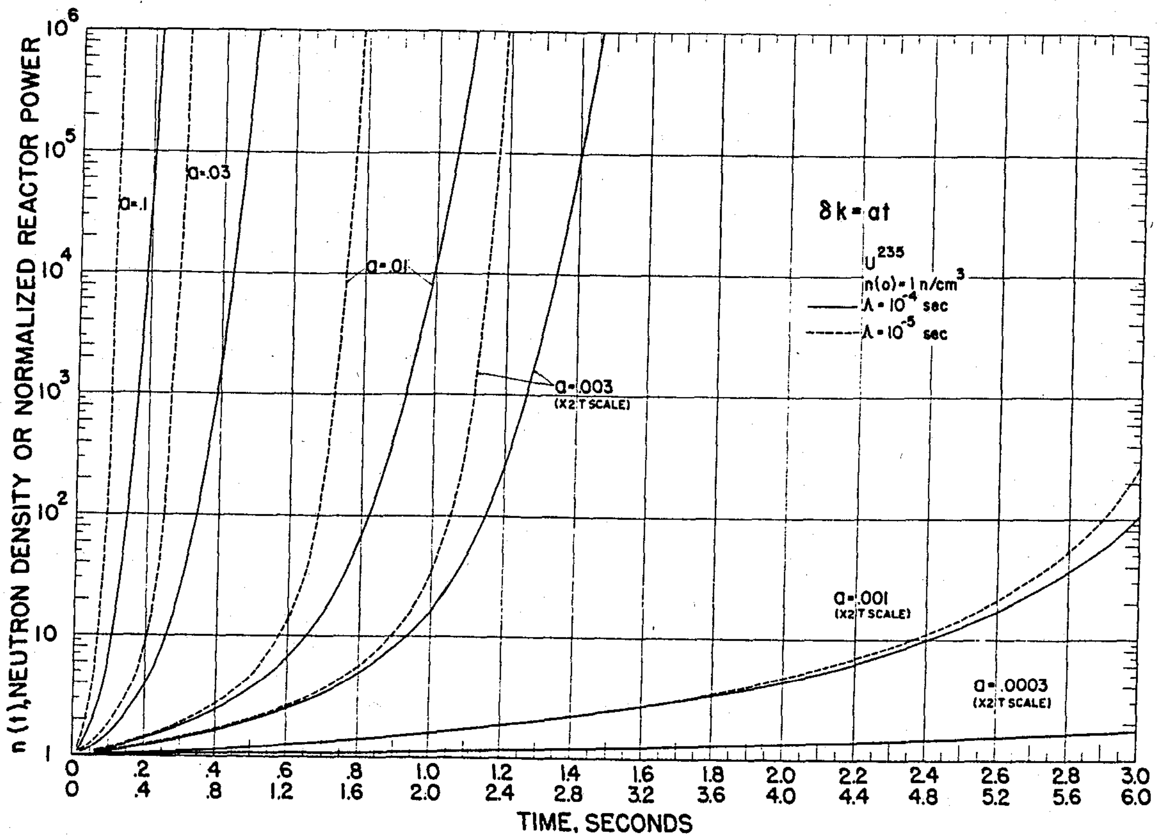


FIG. 1. Response to linear time variation of reactivity in $U^{235}$ systems characterized by prompt neutron generation times in the range $10^{-4}$ sec. to $10^{-5}$ sec. All computations started from initial equilibrium with $n(0) \equiv 1$ neutron/cm³.

feature of the RTS code, whereby sets of data points can be treated in sequence (cf., Appendix A).

In certain reactor control problems, one wishes to determine the $\delta k(t)$ variation required to produce a given $n(t)$; i.e., a specified power distribution as a function of time. In this case (the inverse of the $n(t)$ response calculation) Eq. (7) can be used equally well to generate the $\delta k(t)$ corresponding to a prescribed $n(t)$. The explicit solution for $\delta k_m$ in the $m$th time interval is obvious from inspection of Eq. (7a). (This alternative problem, "given $n(t)$, find $\delta k(t)$," is presently being included as part of the RTS code.)

## COMPENSATED RESPONSE CALCULATIONS

The integral form of solution, Eq. (7), may also be used to calculate self-limiting power excursions, reactor accidents, etc., by including reactivity-compensating effects. Upon introducing the appropriate relation between fission energy release and reactivity, Eq. (7) can be solved numerically for $n(t)$ and/or $\delta k(t)$ given the initial conditions on $\delta k$, $n$, and the $C_i$.

Assuming a shutdown effect proportional to integrated neutron density (which is in turn proportional to fission energy release for given $\Lambda$), we represent $\delta k(t)$ in generalized notation:

$$\delta k(t) = P_1(t) + P_2(t) \int_0^t n(t')\, dt' \qquad (8)$$

Here $P_1(t)$ represents the impressed reactivity variation (generalized polynomial in $t$, or data fit), and $P_2(t)$ is the "shutdown coefficient" of the reactor system. As presently provided in the RTS code, $P_2(t)$ may be a polynomial in $t$ of degree $\leq 49$, although it is usually taken as a negative constant of magnitude $B$, ranging from $\sim 10^{-13}$ cm³/sec for slow systems to $B \sim 10^{-7}$ cm³/sec for fast metal systems.

Typical compensated response calculations using the RTS code are illustrated in Figs. 5–7. The results in Fig. 5 are self-limiting excursions produced by ramp-function additions of reactivity in $U^{235}$-graphite systems characterized by prompt neutron generation times in the region of $5 \times 10^{-5}$ sec, and $B$ values ranging between $10^{-11}$ and $10^{-13}$ cm³/sec. Both $n(t)$ and $\delta k(t)$ variations are plotted in Fig. 5; both exhibit the characteristic damped oscillatory

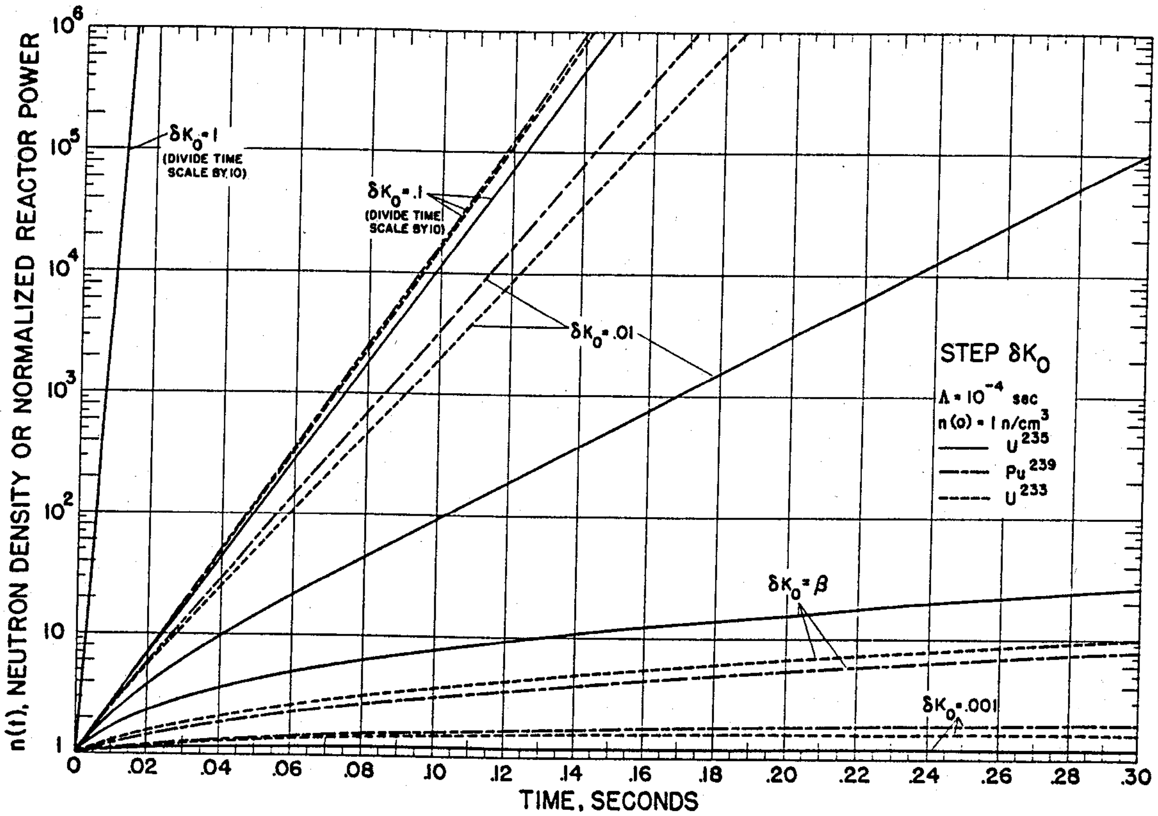


FIG. 2. Response to step changes of reactivity from initial equilibrium in $U^{235}$, $Pu^{239}$, and $U^{233}$ systems; neutron generation time, $\Lambda = 10^{-4}$ sec.

approach to equilibrium power level at which the rate of reactivity compensation due to (adiabatic) temperature increase just balances the rate of external reactivity addition. Heat loss from most practical systems would, of course, result in correspondingly higher equilibrium power levels.[6]

The power peaks in Fig. 5 are in accord with idealized analytic solutions (*9, 10*) of reactivity-compensated response to ramp-function additions of reactivity. Analytic solutions (i.e., neglecting delayed neutrons) are necessarily based on prompt neutron multiplication and hence apply only to systems which are brought to prompt critical in times small compared to the delayed neutron periods.

In compensated response calculations reactivity compensation is often expressed proportional to fission energy density rather than to integrated neutron density. The constant shutdown coefficient, $B$, in units of cm³/sec then becomes $B_\epsilon$ in units of cm³/joule:

$$B_\epsilon = B/v\Sigma_{f\epsilon} \simeq B\Lambda\bar{\nu}/\epsilon \text{ cm}^3/\text{joule}$$

$$\epsilon \simeq 3 \times 10^{-11} \text{ joules/fission}$$

$$\bar{\nu} \equiv \text{average number of neutrons per fission}$$

For the case of compensated step-function response we have

$$\delta k = \delta k_0 - B_\epsilon \int_0^t \dot{E}(t')\, dt' \qquad (9)$$

where $\dot{E}(t')dt' = v\Sigma_{f\epsilon}\, n(t')dt'$ joules/cm³ and $\delta k_0$ is the impressed reactivity step. Figures 6 and 7 were taken from a series of compensated response calculations for various reactivity step functions carried out in the "energy"—rather than the "neutron density"—formulation.[7] An average

[6] In first approximation one may assume a single relaxation constant, $R$, for energy escape from the fissioning core volume, so that the integral term in Eq. (8) would become

$$\int_0^t n(t')e^{-R(t-t')}\, dt' \qquad (8a)$$

Thus heat loss can be accounted for in the RTS code by specifying an appropriate value of $R$ as part of the input data. More sophisticated heat-loss models could be introduced but are not presently included in the RTS code.

[7] Provision is made in the RTS code for printing out power density in units of watts/cm³ and energy density in joules/cm³ as an alternative to the $n(t)$ and $\int n\, dt$ print-out. For both quantities, the conversion factor is $v\Sigma_{f\epsilon} \simeq \epsilon/\Lambda\bar{\nu} \simeq 1.2 \times 10^{-11}\ \Lambda^{-1}$ watts.

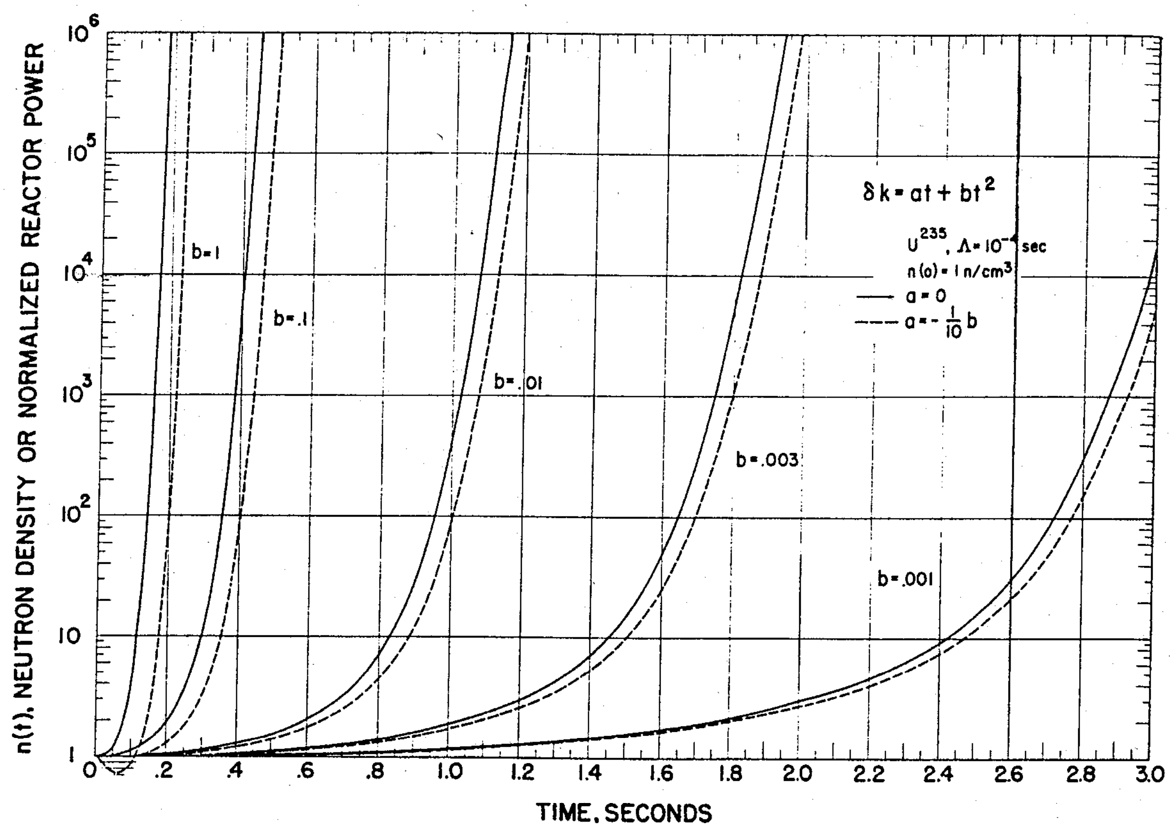


FIG. 3. Response to quadratic time variation of reactivity in $U^{235}$ systems with $\Lambda \simeq 10^{-4}$ sec. All computations started from initial equilibrium with $n(0) \equiv 1$ neutron/cm³.

energy-shutdown coefficient, $B_\epsilon = 2 \times 10^{-5}$ cm³/joule was assumed in both cases. The curve for $\Lambda = 10^{-8}$ in Fig. 6 resembles observed burst characteristics on the Godiva (bare $U^{235}$ metal) assembly at Los Alamos (*11*), while the curves for $\Lambda = 10^{-4}$ and $10^{-6}$ sec are comparable to excursion characteristics observed in intermediate and slow systems (*12*).

Analytic solutions obtained by neglecting the delayed neutrons lead to theoretical dependence of peak power $\sim\frac{1}{2}\delta k_p^2/\Lambda B_\epsilon$, peak width $\sim 4\Lambda/\delta k_p$, total energy release $\sim 2\delta k_p/B_\epsilon$ and plateau height $\sim 1/B_\epsilon$. Each of these relationships has been independently confirmed in various computed response curves such as those plotted in Fig. 6 for $\Lambda = 10^{-6}$ and $10^{-8}$ sec. On the other hand, analytic solutions based exclusively on prompt neutron kinetics do not predict the response to reactivity steps $<$\$1.00, as shown for example in Fig. 7. In this case (\$1.00 step) the departure from characteristic prompt neutron behavior is unmistakable, peak power and peak width being nearly independent of $\Lambda$.[8] Clearly, wherever kinetic behavior is dominated by the delayed neutrons, precise numerical evaluation of reactor transient response becomes especially important.

It is true that for extremely fast transients, governed almost exclusively by the prompt neutrons, exact solutions of the reactor kinetic equations are not always required, or even desired. Indeed in many such cases, approximate analytic solutions (*9–13*), reduced group representations (*14–16*) or other numerical methods (*13, 17–20*) may be preferable. In the delayed-to-prompt-critical regime, however, kinetic behavior is demonstrably sensitive to the delayed neutron characteristics, and the six-group kinetic equations are often required to assure conservative evaluations (*12, 21*).

As an alternative method for solution of the

[8] In Fig. 7 we have assumed an idealized adiabatic system with no heat transfer, no fuel element melting, warping, etc. We see from the $\delta k(t)$ plot that the \$1.00 step is ultimately reflected about *delayed* critical, just as a reactivity step is initially reflected about *prompt* critical in the case of prompt excursions (cf., Fig. 6). Similarly, computed total energy release in Fig. 7 is $\sim 2\delta k_0/B_\epsilon$, just as total energy release in prompt excursions is $2\delta k_p/B_\epsilon$.

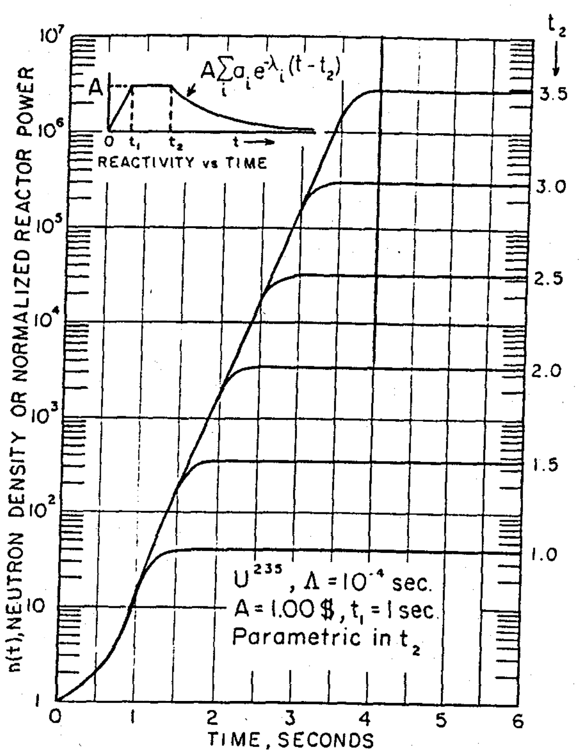


FIG. 4. Response to reactivity variation (see profile in inset; $a_i \equiv \beta_i/\beta$) designed to produce steady power increase to a predetermined constant power level. $\Lambda$ taken as $10^{-4}$ sec. All calculations started from initial equilibrium with $n(0) \equiv 1$ neutron/cm³.

reactor kinetic equations in the delayed-neutron regime, the RTS code offers the simplicity and accuracy of direct numerical evaluation of transient behavior. As suggested by the brief parameter studies outlined here, the code has proved especially useful when extensive parametric survey calculations are required for a wide range of $\delta k(t)$ variations. To indicate the speed of the code, representative computing times for uncompensated and compensated response problems are cited in Appendix A. It might be mentioned that operational experience with the RTS code has been gained in applications to reactor kinetics and control problems associated with the nuclear propulsion (ROVER) program at LASL; checks with analog simulator results were obtained in the course of these studies.

It is intended that this presentation of Eq. (7), and the RTS code, together with the universal constants $A_j$, $B_j$, and $S_j$ given in Tables I, II, and III, will provide a rapid, straightforward method for calculating the detailed kinetic behavior of a wide variety of practical systems.

## APPENDIX A: THE RTS (REACTOR TRANSIENT SOLUTION) CODE

### RECURSION RELATIONS

Equation (7) of the text, which gives neutron density as a function of time, is

$$n(t) = n(0) + \sum_{j=0}^{6} A_j \int_0^t e^{S_j(t-t')} \cdot \delta k(t') n(t')\, dt' + \Omega_0(t) \tag{A-1}$$

The RTS code solves this equation recursively at each time point for reactivities of the form

$$\delta k(t) = P_1(t) + P_2(t) \int_0^t e^{R(t-t')}\, n(t')\, dt' \tag{A-2}$$

where $P_1(t)$ is either (1) a polynomial in $t$ of specified degree, or (2) a table of data pairs $(t_i, \delta k_i)$ to be fitted by the code using Aitken's interpolation method. The number of data pairs to be used and the desired degree of fit must be specified as input parameters.

$P_2(t)$ is a polynomial in $t$ of specified degree. For $P_2(t)$ identically zero the feedback term in (A-2) vanishes, and one may develop a linear solution to Eq. (A-1).

Introducing the integration time interval, $h$, neutron density at time $t + 2h$ may be written

$$n(t+2h) = n(0) + \sum_{j=0}^{6} A_j \int_0^{t+2h} e^{S_j(t+2h-t')} \cdot \delta k(t') n(t')\, dt' + \Omega_0(t+2h)$$

$$= n(0) + \sum_{j=0}^{6} \left\{ \begin{array}{l} A_j \int_0^t e^{S_j(t+2h-t')}\, \delta k(t') n(t')\, dt' \\ + A_j \int_t^{t+2h} e^{S_j(t+2h-t')}\, \delta k(t') n(t')\, dt' \end{array} \right\} + \Omega_0(t+2h)$$

If we define

$$I_j(t) \equiv A_j \int_0^t e^{S_j(t-t')}\, \delta k(t') n(t') dt'$$

for all $j$ and $t$, we have

$$n(t+2h) = n(0) + \sum_{j=0}^{6} I_j(t+2h) + \Omega_0(t+2h)$$

$$= n(0) + \sum_{j=0}^{6} \left\{ e^{S_j 2h} I_j(t) + A_j \int_t^{t+2h} e^{S_j(t+2h-t')}\, \delta k(t') n(t')\, dt' \right\} + \Omega_0(t+2h)$$

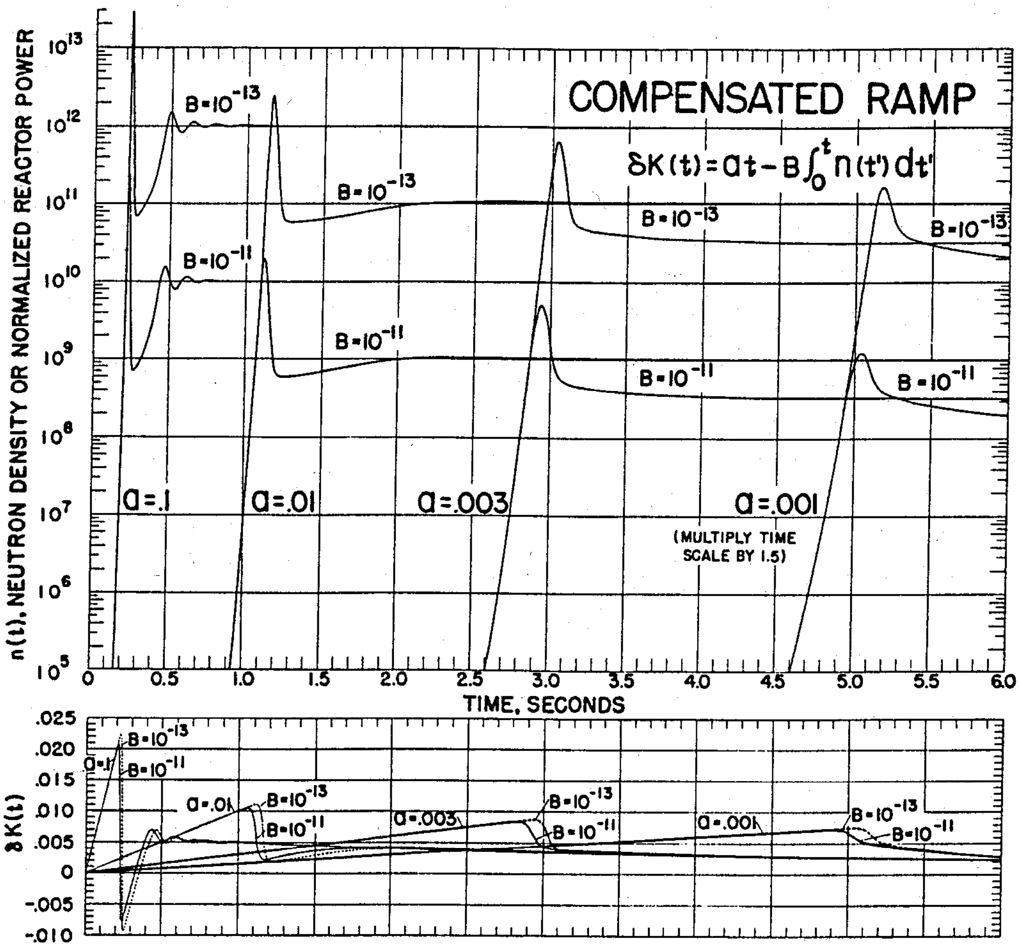


FIG. 5. Compensated response to ramp function reactivity changes in $U^{235}$ systems with prompt neutron generation time, $\Lambda = 5 \times 10^{-5}$ sec, and shutdown coefficients in the range $B = 10^{-11}$ cm³/sec to $10^{-13}$ cm³/sec. All calculations started from initial equilibrium with $n(0) \equiv 1$ neutron/cm³.

Using Simpson's rule[9] for integration, we obtain the linear expressions for $n(t+2h)$ and $I_j(t+2h)$ in terms of $n(t)$, $n(t+h)$, $n(t+2h)$, and $I_j(t)$:

$$n(t+2h) \cong n(0) + \sum_{j=0}^{6} e^{S_j 2h} I_j(t) + hA_j \left\{ \begin{array}{l} W_0\, e^{S_j 2h}\, \delta k(t) n(t) \\ + W_1\, e^{S_j h}\, \delta k(t+h) n(t+h) \\ + W_2\, \delta k(t+2h) n(t+2h) \end{array} \right\} + \Omega_0(t+2h) \qquad \text{(A-3)}$$

$$I_j(t+2h) \cong e^{S_j 2h} I_j(t) + hA_j \left\{ \begin{array}{l} W_0\, e^{S_j 2h} \delta k(t) n(t) \\ + W_1\, e^{S_j h} \delta k(t+h) n(t+h) \\ + W_2\, \delta k(t+2h) n(t+2h) \end{array} \right\} \qquad \text{(A-4)}$$

where $W_0 = W_2 = \frac{1}{3}$ and $W_1 = \frac{4}{3}$.

To obtain linear expressions for $n(t+h)$ and $I_j(t+h)$ in terms of $n(t)$, $n(t+h)$, and $I_j(t)$, one follows a similar procedure. Using the trapezoidal rule[10] for integration this leads to the relations

$$n(t+h) \cong n(0) + \sum_{j=0}^{6} e^{S_j h} I_j(t) + hA_j \left\{ \begin{array}{l} \frac{1}{2}\, e^{S_j h}\, \delta k(t) n(t) \\ + \frac{1}{2}\, \delta k(t+h) n(t+h) \end{array} \right\} + \Omega_0(t+h) \qquad \text{(A-5)}$$

[9] $\int_a^b y(x)\, dx \cong \frac{1}{2}(b-a)\left[\frac{1}{3} y(a) + \frac{4}{3} y \frac{(a+b)}{2} + \frac{1}{3} y(b)\right]$.

[10] $\int_a^b y(x)\, dx \cong \frac{1}{2}(b-a)[y(a) + y(b)]$.

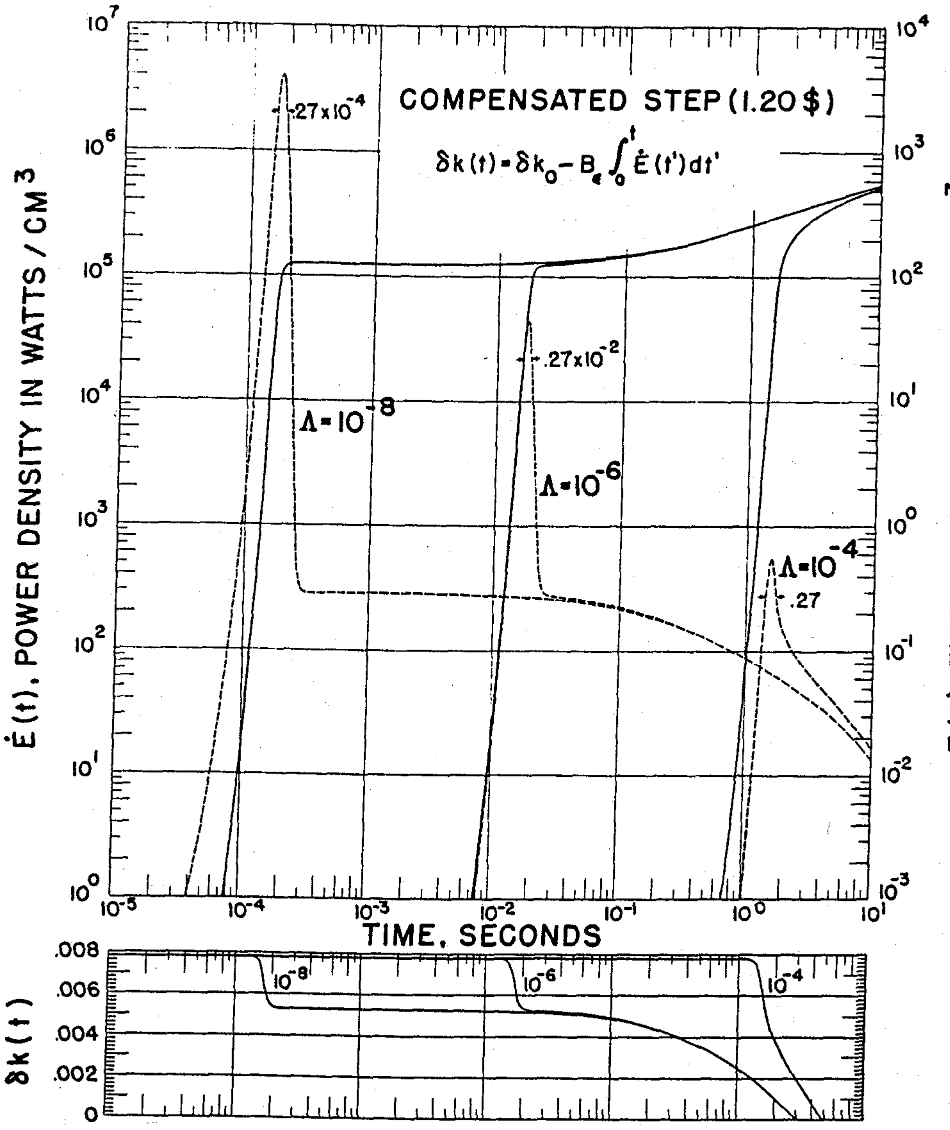


FIG. 6. Compensated response to a step reactivity change of \$1.20 ($\delta k_0 = 0.0078$) in $U^{235}$ systems characterized by an energy shutdown coefficient, $B_\epsilon = 2 \times 10^{-5}$ cm³/joule, and representative neutron generation times $\Lambda$, $= 10^{-8}$, $10^{-6}$ and $10^{-4}$ sec. Dotted curves are power response and solid curves are energy response. Initial neutron density, $n(0) \equiv 1$ neutron/cm³ in all cases.

$$I_j(t+h) \cong e^{S_j h} I_j(t) + hA_j \left\{ \begin{matrix} \frac{1}{2}\, e^{S_j h}\, \delta k(t) n(t) \\ + \delta k(t+h) n(t+h) \end{matrix} \right\} \quad \text{(A-6)}$$

Simpson's rule for integration is usually more accurate than the trapezoidal rule and is used whenever possible in the RTS code. The expressions for $n(t+h)$ and $I_j(t+h)$ which involve the trapezoidal rule are used only to obtain values to start the recursion. [If desired, trapezoidal integration can be used throughout the computation by setting $W_0 = W_2 = \frac{1}{2}$ and $W_1 = 1$ in (A-3) and (A-4).]

To start the recursion one needs values for $n(t)$ and $I_j(t)$. In particular when $t = 0$, $n(t)$ is known and $I_j(t) \equiv 0$. One then uses (A-5) and (A-6) to obtain values for $n(h)$ and $I_j(h)$. With values for $n(0)$, $n(h)$, and $I_j(0)$ one obtains values for $n(2h)$ and $I_j(2h)$ from (A-3) and (A-4). The recursion is continued using (A-3) and (A-4).

In actual practice the RTS code computes $n(h/2)$ by trapezoidal integration making it possible to compute $n(h)$ by Simpson's rule. The recursion relations (A-3) and (A-4) are used for subsequent time intervals.

For compensated-reactivity problems, where $P_2(t)$ is of degree $>0$ or is a constant $\neq 0$, the intro-

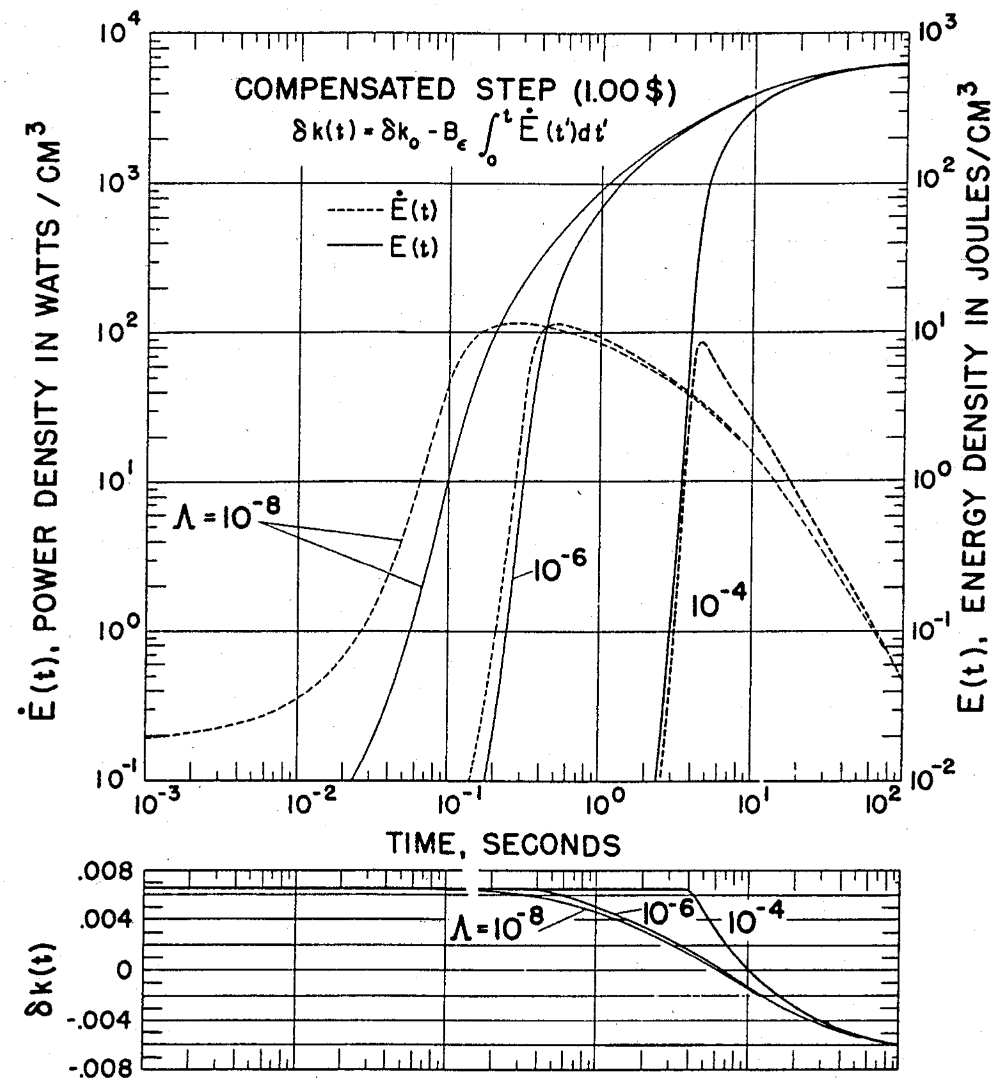


FIG. 7. Compensated response to a step reactivity change of \$1.00 ($\delta k_0 = 0.0065$) in $U^{235}$ systems characterized by an energy shutdown coefficient, $B_\epsilon = 2 \times 10^{-5}$ cm³/joule, and representative neutron generation times, $\Lambda \equiv 10^{-8}$, $10^{-6}$ and $10^{-4}$ sec. Initial neutron density, $n(0) \equiv 1$ neutron/cm³ in all cases.

duction of a feedback relation between $\delta k(t)$ and $n(t)$ leads to a quadratic expression for $n(t)$ which must be solved at each time point. If one defines

$$I_R(t) \equiv \int_0^t e^{R(t-t')} n(t')\, dt'$$

expression (A-2) at time $t + 2h$ becomes

$$\delta k(t + 2h) = P_1(t + 2h) + P_2(t + 2h)\, I_R(t + 2h) \qquad \text{(A-7)}$$

Using Simpson's rule and following the same procedure as for $I_j(t + 2h)$ in (A-4),

$$I_R(t + 2h) \cong e^{R2h}\, I_R(t) + h \left\{ \begin{array}{ll} W_0\, e^{R2h} & n(t) \\ +W_1\, e^{Rh} & n(t + h) \\ +W_2 & n(t + 2h) \end{array} \right\}$$

Hence (A-7) becomes

$$\delta k(t + 2h) \cong P_1(t + 2h) + P_2(t + 2h) \cdot \left[ e^{R2h} I_R(t) + h \left\{ \begin{array}{ll} W_0 e^{R2h} & n(t) \\ +W_1 e^{Rh} & n(t + h) \\ +W_2 & n(t + 2h) \end{array} \right\} \right]$$

and (A-3) becomes a quadratic expression in $n$:

$$C_0(t + 2h) n^2(t + 2h) + C_1(t + 2h) n(t + 2h) + C_2(t + 2h) \cong 0 \qquad \text{(A-8)}$$

where

$$C_0(t + 2h) = -(hW_2)^2 \sum_{j=0}^{6} A_j P_2(t + 2h)$$

$$C_1(t + 2h) = 1 - (hW_2) \sum_{j=0}^{6} A_j \Big[ P_1(t + 2h)$$

$$+ P_2(t + 2h)\left(e^{R2h}I_R(t) + h\begin{Bmatrix} W_0\, e^{R2h} n(t) \\ + W_1\, e^{Rh}\, n(t+h)\end{Bmatrix}\right)\Bigg]$$

$$C_2(t + 2h) = -n(0) - \sum_{j=0}^{6}\Bigg[e^{S_j 2h} I_j(t) + hA_j\begin{Bmatrix} W_0 e^{S_j 2h}\delta k(t) n(t) \\ + W_1 e^{S_j h}\delta k(t+h) n(t+h)\end{Bmatrix}\Bigg] - \Omega_0(t + 2h)$$

Experience with the RTS code thus far has shown that

$$0 < C_0(t) \ll 1$$
$$C_1(t) \simeq 1$$
$$C_2(t) \simeq -n(t)$$

The explicit solution of Eq. (A-8) leads to loss of significant figures for values of $n$ near unity. Newton's iteration method[11] is used to reduce this possible loss of figures:

$$n_{i+1} = n_i - \frac{[C_0 n_i^2 + C_1 n_i + C_2]}{2C_0 n_i + C_1} = \frac{C_0 n_i^2 - C_2}{2C_0 n_i + C_1}$$

Initially $n_i$ is set equal to $-C_2/C_1$; iteration on $n$ is then continued until

$$|(n_{i+1} - n_i)/n_i| < \epsilon$$

where $\epsilon$ is specified $\sim 10^{-7}$.

Having thus determined $n(t)$, values of the functions $n$, $C_0$, $C_1$, and $C_2$ at succeeding time intervals, $t + h$ and $t + 2h$ are generated recursively as indicated above.

Similar recursion relations are developed for $\Omega_0(t)$ when this quantity is nonzero (nonequilibrium case), viz:

$$\Omega_0(t + 2h) = \sum_{j=0}^{6}\Bigg[e^{S_j 2h} I_{\Omega_j}(t) + hB_j\begin{Bmatrix} W_0 e^{S_j 2h} G(t) \\ + W_1 e^{S_j h} G(t+h) \\ + W_2 G(t + 2h)\end{Bmatrix}\Bigg]$$

with

$$I_{\Omega_j}(t) \equiv B_j \int_0^t e^{S_j(t-t')} G(t')\, dt'$$

$$G(t) \equiv \sum_{i=1}^{6} [\lambda_i C_i(0) - \Lambda^{-1}\beta_i n(0)]\gamma_i e^{-\lambda_i t} + \gamma_S S(t)$$

[11] $x_{i+1} = x_i - [f(x_i)/f'(x_i)]$.

VARIABLE INTEGRATION TIME INTERVAL

In the RTS code, the integration time interval, $DT \equiv h$, can be varied automatically in an attempt to optimize computing speed and accuracy. The factor by which $h$ is changed ($DT$ MULT) is an input variable, normally specified in the range 1 to 2. The magnitude of the relative change in $n$ per time interval $|\delta n/n|$, is the criterion used to dictate $h$ changes. The test sequence used in the code is as shown below.

If

| | |
|---|---|
| $\left\lvert\frac{\delta n}{n}\right\rvert < F_0$ | $h$ is set equal to previous $h$ and: (option 1), testing is continued at each time point (option 2), $h$ is held constant for the remainder of the run without further testing |
| $F_0 \leqq \left\lvert\frac{\delta n}{n}\right\rvert < F_1$ | $h$ is increased by the factor, $DT$ MULT |
| $F_1 \leqq \left\lvert\frac{\delta n}{n}\right\rvert < F_2$ | $h$ is unchanged |
| $F_2 \leqq \left\lvert\frac{\delta n}{n}\right\rvert$ | $h$ is decreased by the factor, $DT$ MULT |

Typical parameter values are $DT$ MULT $= 1.2$, $F_0 = 10^{-4}$, $F_1 = 3 \times 10^{-4}$, $F_2 = 3 \times 10^{-3}$. The $F_0$ fiducial was introduced to minimize oscillations in $n(t)$ and $\alpha(t)$ which were found likely to develop during variable $h$ operation when $|\delta n/n|$ becomes very small. To hold $h$ constant one sets $F_0 = 10^{38}$ (in which case values of $F_1$ and $F_2$ are immaterial), and the input value of $h$ will be maintained throughout the problem.

INPUT-OUTPUT FEATURES

Quantities to be specified as input data to the RTS code are

Form of $P_1(t)$:
(1) Polynomial: specify degree, and coefficients
(2) Data fit: specify degree of polynomial fit, and data pairs, $t_i$, $\delta k_i$, $i \leqq 500$.

Form of $P_2(t)$: Polynomial: specify degree and coefficients. Specify:

$$\beta,\ \Lambda,\ A_j,\ S_j(j = 0, 6), \bar{\nu},$$
$$DT,\ DT\ \text{MULT},\ F_0,\ F_1,\ F_2, \text{ and } R$$

If $\Omega_0(t) = 0$, specify:

$\Omega$ INDIC $\equiv 0$, and omit all parameters specified below for $\Omega_0(t) \neq 0$

If $\Omega_0(t) \neq 0$, specify:

$\Omega$ INDIC $\equiv 1, \lambda_i, \beta_i, C_i(0), \gamma_i(i = 1, 6)$; $\gamma_S$, Define $S(t)$ and supply needed parameters;

$$B_j(j = 0, 6)$$

"Stop-Start" Feature

Input values must be specified for $t$, $n$, $I \equiv \int n\, dt$, $I_R$, $C_0$, $C_1$, $C_2$,

$$I_j;\quad \text{if}\quad \Omega_0(t) \neq 0,\quad \text{specify } I_{\Omega_j}$$

At the end of each run, current values of these functions (and the current integration time interval, $DT$) are punched on-line onto cards and written on tape for off-line printing. This OUTPUT DUMP can then be used directly to continue the problem, in which case these function values are again written on tape to be printed as INPUT DUMP values for the continuation run.

A run can be stopped manually at any time (sense switch 1 down). Alternatively, current information can be printed on-line at any desired time (by depressing sense switch 2) and the program continued.

Print Data

Specify:

- $ND$, the number of integrations for each point printed
- MAX $PRT$, number of points to be printed in a run, $\leqq 10^3$
- $T$ MAX, maximum (termination) value of $t$ in a run
- $N$ MAX, maximum permissible value of $n$ in a run
- COMMENT, optional comment may be loaded on one card to identify problem.

Labeled quantities included in the RTS print-out format [print options in brackets] are Heading; Comment statement; Input parameters; Input dump; $t$; $n(t)$, [watts/cm$^3$]; $\int ndt$, [joules/cm$^3$]; $\delta k(t)$, [$S(t)$];$DT$; $\Omega_0(t)$; $C_0(t)$; $C_1(t)$; $C_2(t)$; $t_D \equiv \alpha$ time base; $\alpha(t_D) \equiv \dot{n}(t_D)/n(t_D)$; Output Dump.

With regard to computing speed: for constant time intervals, $h$, the computation proceeds at the rate of ~1500 integrations per minute, this rate decreasing slightly when $h$ is allowed to vary. Integration time intervals are initially the order of $\Lambda/\beta$ (maximum) for step reactivities; generally larger for ramps and continuous $\delta k$ functions. Representative computing times are ~10 min per curve for uncompensated problems as in Figs. 1–4; ~30 min per curve for compensated problems as in Figs. 5–7.

For further details, reference is made to the RTS program listing and flow diagram on file in the LASL code library.

Acknowledgments

We wish to thank T. L. Jordan, B. G. Carlson, and G. I. Bell for their support and interest in this work. The assistance of L. Bordwell in programming RTS computations on the IBM 704 computer is gratefully acknowledged.

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